Chapter 6

Decimation in Time

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In this method we split \( x(n) \) into the even indexed \( x(2m) \) and the odd indexed \( x(2m + 1) \) each \( N/2 \) long. DFT can be expressed as:

\[
X(k) = \sum_{m=0}^{N/2-1} x(2m)W_N^{2mk} + \sum_{m=0}^{N/2-1} x(2m + 1)W_N^{k(2m+1)}
\]

\[
X(k) = \sum_{m=0}^{N/2-1} x(2m)W_N^{mk}W_N^{k/2} + W_N^{k} \sum_{m=0}^{N/2-1} x(2m + 1)W_N^{km}N/2
\]

\[K = 0, 1, 2, 3, 4, \ldots \ldots \ldots, N-1\]

We have used the fact

\[W_N^{2mk} = e^{-j2mk \times 2\pi / N} = e^{-j2\pi mk/(N/2)} = W_N^{km_{N/2}}\]
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Define new functions as

\[ G(k) = \sum_{m=0}^{N/2-1} x(2m)W_{N/2}^{mk} \quad k = 0,1,2,\ldots,N/2-1 \]

\[ H(k) = \sum_{m=0}^{N/2-1} x(2m+1)W_{N/2}^{km} \quad k = 0,1,2,3,\ldots,N/2-1 \]

Then equation (1) for the first \( N/2 \) points can be expressed as

\[ X(k) = G(k) + W_N^k H(k) \quad k = 0,1,2,\ldots,N/2-1 \]
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But

\[ W_N^{(N/2+k)} = -W_N^k \]

\[ G(k) = G(N/2 + k), \quad k = 0, 1, 2, ..., N/2 - 1 \]

\[ H(k) = H(k + N/2), \quad k = 0, 1, 2, ..., N/2 - 1 \]

Then

\[ X(k + N/2) = G(k) - W_N^k H(k), \quad k = N/2, N/2 + 1, ..., N \]

This can be expressed in the following diagram
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4-point DFT

\begin{align*}
    x(0) & \rightarrow G(0) \rightarrow X(0) \\
    x(2) & \rightarrow G(1) \rightarrow X(1) \\
    x(4) & \rightarrow G(2) \rightarrow X(2) \\
    x(6) & \rightarrow G(3) \\
    x(1) & \rightarrow H(0) \rightarrow X(4) \\
    x(3) & \rightarrow H(1) \rightarrow X(5) \\
    x(5) & \rightarrow H(2) \rightarrow X(6) \\
    x(7) & \rightarrow H(3) \rightarrow X(7)
\end{align*}

Digital Signal Processing
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Decimation in Time is a process used in digital signal processing to reduce the sampling rate of a discrete-time signal. It involves the selection of every Nth sample from an input sequence, where N is the decimation factor. In the diagram, the input sequence $x(n)$ is processed through a series of 2-point DFT (Discrete Fourier Transform) blocks. Each block computes the Fourier transform of a subset of the input sequence, effectively reducing the sampling rate.

The diagram shows the relationship between the input samples $x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7)$ and the output samples $X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)$. The process involves multiplying the input samples by the twiddle factors $W^0_8, W^2_8, W^0_8, W^2_8, W^0_8, W^2_8, W^0_8, W^2_8$ respectively, before applying time-domain filtering with the factor $-1$. The output samples are obtained by summing the filtered results.
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\[ X(0) \]
\[ X(1) \]
\[ X(2) \]
\[ X(3) \]
\[ X(4) \]
\[ X(5) \]
\[ X(6) \]
\[ X(7) \]

\[ W_8^0 \]
\[ W_8^0 \]
\[ W_8^0 \]
\[ W_8^0 \]
\[ W_8^0 \]
\[ W_8^0 \]
\[ W_8^0 \]

\[ W_8^2 \]
\[ W_8^2 \]
\[ W_8^2 \]
\[ W_8^2 \]
\[ W_8^2 \]
\[ W_8^2 \]
\[ W_8^2 \]
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Example Given a sequence $x(n)$ where $x(0) = 1$, $x(1) = 2$, $x(2) = 3$, $x(3) = 4$ and $x(n) = 0$ elsewhere, find DFT for the first four points.

Solution

\[
\begin{align*}
X(0) &= 10 \\
X(1) &= -2 + 2j \\
X(2) &= -2 \\
X(3) &= -2 - 2j
\end{align*}
\]
Inverse Fourier Transform

The inverse discrete Fourier can be calculated using the same method but after changing the variable $W_N$ and multiplying the result by $1/N$

**Example** Given a sequence $X(n)$ given in the previous example. Find the IFFT using decimation in time method

**Solution**

\[
\begin{align*}
X(0) &= 10 \\
X(2) &= -2 \\
X(1) &= -2 + 2j \\
X(3) &= -2 - 2j
\end{align*}
\]