Chapter 2

Discrete-Time Signal (DTS)

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Representation of DTS

1) Function Representation

\[ x(n) = \begin{cases} 
1 & \text{for } n = 1,3 \\
4 & \text{for } n = 2 \\
0 & \text{elsewhere} 
\end{cases} \]

2) Graphical representation

3) Tabular Representation

<table>
<thead>
<tr>
<th>n</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(n)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Digital Signal Processing
Representation of DTS

4) Sequence Representation

An infinite signal with time origin \( n = 0 \) marked by (↑)
can be represented as a sequence
\[ X(n) = \{\ldots 0.1.4.1.1.0.0.\ldots\} \]

If \( x(n) \) is zero for \( n < 0 \)
\[ X(n) = \{0.1.4.1.0.0.\ldots\} \]

A finite duration sequence \( x(n) \) can be represented by:
\[ X(n) = \{3,-1,-2,5,0,4,-1\} \]

Elementary DTS

1) Unit Sample (impulse)

\[ \delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{elsewhere} \end{cases} \]

2) Unit Step

\[ u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases} \]
3) Unit ramp signal

\[ u_r(n) = \begin{cases} 
  n & \text{for } n \geq 0 \\
  0 & \text{for } n < 0
\end{cases} \]

4) Exponential Signal

\[ x(n) = a^n \]

- \( 0 < a < 1 \)
- \( a > 1 \)
- \( -1 < a < 0 \)
- \( a < -1 \)
Classification of Discrete Time Signals

1) Energy Signal and Power signal

The energy $E$ of a signal $x(n)$ is defined as

$$E = \sum_{n = -\infty}^{\infty} |x(n)|^2$$

Example: Energy of the shown signal $E = 1 + 1 + 4 \times 2 = 18$

Energy Signals and Power signals

If $E$ is finite (as in the previous example), the signal is called an energy signal.

If $E$ is infinite then the power of the signal can be calculated as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

- Note for finite energy, signal power equals zero.
- If $E$ is infinite $P$ can be finite or infinite.
- If $P$ is finite then signal is called power signal.
Classification of Discrete Time Signals

Energy Signals and Power signals

Example: Determine the power and energy of unit step sequence

\[ E \text{ is infinite, then the power equals} \]
\[ P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} |u(n)|^2 \]
\[ P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 = \lim_{N \to \infty} \frac{N+1}{2N+1} \]
\[ P = \lim_{N \to \infty} \frac{1+1/N}{2+1/N} = 1/2 \]

P is finite, then unit step is a power signal

2) Periodic and not periodic signals

\( x(n) \) is periodic if:
\[ x(n+N) = x(n) \]

The smallest value of \( N \) is called fundamental period
Classification of Discrete Time Signals

3) Even and odd signals

Even signal

\[ x(n) \text{ is even if: } x(-n) = x(n) \]

\[ x(n) \text{ is even if: } x(-n) = -x(n) \]

Odd signal

Exercise: sketch the even and odd component for the unit step function?
**Simple Manipulations of DTS**

1) **Time shift**

- $x(n-k)$ is a shifted version of $x(n)$
- If $k$ is positive, $x(n-k)$ is a delayed version of $x(n)$
- If $k$ is negative, $x(n-k)$ is an advanced version of $x(n)$

![Time shift examples](image)

2) **Reflection**

- $x(-n)$ is obtained by folding the signal around the time origin ($n=0$)
- Note: to find $x(-n-k)$, you need to do the reflection first then the shift.

- Considering the shift of $x(-n)$ would be on the opposite direction to the shift of $x(n)$
  - $K$ positive: shift $x(-n)$ to the left
  - $K$ negative shift $x(-n)$ to the right

- Exercise: for the shown signal find $x(-n+4)$

![Reflection examples](image)
### Simple Manipulations of DTS

3) Scaling

$X(\mu n)$ is called down sampling, where $\mu$ is an integer

\[
y(n) = x(2n)
y(0) = x(0)
y(1) = x(2)
y(2) = x(4)
\]

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### Homework

Students are encouraged to solve the following questions from the textbook

2.1.2.2, and 2.4