Chapter (3)
Ultimate Bearing Capacity of Shallow Foundations
**Introduction**

To perform satisfactorily, shallow foundations must have two main characteristics:

1. They have to be safe against **overall shear failure** in the soil that supports them.
2. They cannot undergo excessive displacement, or excessive settlement.

**Note:** The term excessive settlement is relative, because the degree of settlement allowed for a structure depends on several considerations.

**Types of Shear Failure**

Shear Failure: Also called “Bearing capacity failure” and it’s occur when the shear stresses in the soil exceed the shear strength of the soil.

**There are three types of shear failure in the soil:**

1. General Shear Failure
The following are some characteristics of general shear failure:

✓ Occurs over dense sand or stiff cohesive soil.
✓ Involves total rupture of the underlying soil.
✓ There is a continuous shear failure of the soil from below the footing to the ground surface (solid lines on the figure above).
✓ When the (load / unit area) plotted versus settlement of the footing, there is a distinct load at which the foundation fails ($Q_u$).
✓ The value of ($Q_u$) divided by the area of the footing is considered to be the ultimate bearing capacity of the footing ($q_u$).
✓ For general shear failure, the ultimate bearing capacity has been defined as the bearing stress that causes a sudden catastrophic failure of the foundation.
✓ As shown in the above figure, a general shear failure ruptures occur and pushed up the soil on both sides of the footing (In laboratory).
✓ However, for actual failures on the field, the soil is often pushed up on only one side of the footing with subsequent tilting of the structure as shown in figure below:
2. Local Shear Failure:

The following are some characteristics of local shear failure:

- Occurs over sand or clayey soil of medium compaction.
- Involves rupture of the soil only immediately below the footing.
- There is soil bulging (انتشار أو بروز) on both sides of the footing, but the bulging is not as significant as in general shear. That’s because the underlying soil compacted less than the soil in general shear.
- The failure surface of the soil will **gradually** (not sudden) extend outward from the foundation (not the ground surface) as shown by **solid lines** in the above figure.
So, local shear failure can be considered as a transitional phase between general shear and punching shear.

Because of the transitional nature of local shear failure, the ultimate bearing capacity could be defined as the first failure load \( (q_{u,1}) \) which occurs at the point which have the first measure nonlinearity in the load/unit area-settlement curve (open circle), or at the point where the settlement starts rapidly increase \( (q_u) \) (closed circle).

This value of \( (q_u) \) is the required (load/unit area) to extend the failure surface to the ground surface (dashed lines in the above figure).

In this type of failure, the value of \( (q_u) \) is not the peak value so, this failure called (Local Shear Failure).

The actual local shear failure in field is proceed as shown in the following figure:

3. Punching Shear Failure:
The following are some characteristics of punching shear failure:

- Occurs over fairly loose soil.
- Punching shear failure does not develop the distinct shear surfaces associated with a general shear failure.
- The soil outside the loaded area remains relatively uninvolved and there is a minimal movement of soil on both sides of the footing.
- The process of deformation of the footing involves compression of the soil directly below the footing as well as the vertical shearing of soil around the footing perimeter.
- As shown in figure above, the \((q)\)-settlement curve does not have a dramatic break, and the bearing capacity is often defined as the first measure nonlinearity in the \((q)\)-settlement curve \((q_{u,1})\).
- Beyond the ultimate failure (load/unit area) \((q_{u,1})\), the (load/unit area)-settlement curve will be steep and practically linear.
- The actual punching shear failure in field is proceed as shown in the following figure:
Ultimate Bearing Capacity ($q_u$)

It’s the **minimum load per unit area** of the foundation that causes shear failure in the underlying soil.

Or, it’s the **maximum load per unit area** of the foundation can be resisted by the underlying soil without occurs of shear failure (if this load is exceeded, the shear failure will occur in the underlying soil).

Allowable Bearing Capacity ($q_{all}$)

It’s the load per unit area of the foundation can be resisted by the underlying soil without any unsafe movement occurs (shear failure) and if this load is exceeded, the shear failure will not occur in the underlying soil till reaching the ultimate load.

Terzaghi’s Bearing Capacity Theory

Terzaghi was the first to present a comprehensive theory for evaluation of the ultimate bearing capacity of rough shallow foundation. **This theory is based on the following assumptions:**

1. The foundation is considered to be sallow if ($D_f \leq B$).
2. The foundation is considered to be strip or continuous if ($\frac{B}{L} \to 0.0$).

(Width to length ratio is very small and goes to zero), and the derivation of the equation is to a strip footing.

3. The effect of soil above the bottom of the foundation may be assumed to be replaced by an equivalent surcharge ($q = \gamma \times D_f$). So, the shearing resistance of this soil along the failure surfaces is neglected (Lines ab and cd in the below figure)

4. The failure surface of the soil is similar to **general shear failure** (i.e. equation is derived for general shear failure) as shown in figure below.

**Note:**

1. In recent studies, investigators have suggested that, foundations are considered to be shallow if [$D_f \leq (3 \rightarrow 4)B$], otherwise, the foundation is deep.
2. Always the value of ($q$) is the **effective stress** at the bottom of the foundation.
Terzaghi’s Bearing Capacity Equations
As mentioned previously, the equation was derived for a strip footing and general shear failure, this equation is:

$q_u = cN_c + qN_q + 0.5B\gamma N_\gamma$  (for continuous or strip footing)

Where
$q_u =$ Ultimate bearing capacity of the underlying soil (KN/m$^2$)  
$c =$ Cohesion of underlying soil (KN/m$^2$)  
$q =$ Effective stress at the bottom of the foundation (KN/m$^2$)  
$N_c, N_q, N_\gamma =$ Bearing capacity factors (nondimensional) and are functions only of the underlying soil friction angle, $\phi$, $\rightarrow$

The variations of bearing capacity factors and underlying soil friction angle are given in (Table 3.1, P.139) for general shear failure.

The above equation (for strip footing) was modified to be useful for both square and circular footings as following:

For square footing:
$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$
$B =$ The dimension of each side of the foundation .

For circular footing:
$q_u = 1.3cN_c + qN_q + 0.3B\gamma N_\gamma$
$B =$ The diameter of the foundation .

Note:
These two equations are also for general shear failure, and all factors in the two equations (except, $B,$) are the same as explained for strip footing.
Now for **local shear failure** the above three equations were modified to be useful for local shear failure as following:

\[ q_u = \frac{2}{3} c N'_c + q N'_q + 0.5 B \gamma N'_\gamma \]  
(for continuous or strip footing)

\[ q_u = 0.867 c N'_c + q N'_q + 0.4 B \gamma N'_\gamma \]  
(for square footing)

\[ q_u = 0.867 c N'_c + q N'_q + 0.3 B \gamma N'_\gamma \]  
(for circular footing)

\[ N'_c, N'_q, N'_\gamma = \text{Modified bearing capacity factors} \]  
and could be determined by the following **two methods**:

1. **(Table 3.2 P.140)** variations of modified bearing capacity factors and **underlying** soil friction angle.

2. **(Table 3.1 P.139)** (if you don’t have Table 3.2), variation of bearing capacity factors and **underlying** soil friction angle, **but you must** do the following **modification** for the underlying soil friction angle:

\[ \tan \phi (\text{General Shear}) = \frac{2}{3} \times \tan \phi (\text{Local Shear}) \rightarrow \]

\[ \phi_{\text{modified, general}} = \tan^{-1} \left( \frac{2}{3} \tan \phi_{\text{local}} \right) \]

**For example:** Assume we have local shear failure and the value of \( \phi = 30^\circ \)

1. By using (Table 3.2) \( N'_c, N'_q, N'_\gamma = 18.99, 8.31, \) and 4.9 respectively

2. By using (Table 3.1) \( \rightarrow \phi_{\text{modified, general}} = \tan^{-1} \left( \frac{2}{3} \tan 30^\circ \right) = 21.05^\circ \rightarrow \)

\( (N'_c, N'_q, N'_\gamma)_{21.05, \text{table 3.1}} \approx (N'_c, N'_q, N'_\gamma)_{30, \text{table 3.2}} = 18.92, 8.26, \) and 4.31 respectively

**General Bearing Capacity Equation (Meyerhof Equation)**

Terzaghi’s equations shortcomings:

- They don’t deal with rectangular foundations (\( 0 < \frac{B}{L} < 1 \)).

- The equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation (as mentioned previously).

- The inclination of the load on the foundation is not considered (if exist).
To account for all these shortcomings, Meyerhof suggested the following form of the general bearing capacity equation:

\[
q_u = c N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + 0.5 \gamma N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}
\]

Where
- \(c\) = Cohesion of the underlying soil
- \(q\) = Effective stress at the level of the bottom of the foundation.
- \(\gamma\) = unit weight of the underlying soil
- \(B\) = Width of footing (= diameter for a circular foundation).
- \(N_c, N_q, N_\gamma\) = Bearing capacity factors (will be discussed later).
- \(F_{cs}, F_{qs}, F_{\gamma s}\) = Shape factors (will be discussed later).
- \(F_{cd}, F_{qd}, F_{\gamma d}\) = Depth factors (will be discussed later).
- \(F_{ci}, F_{qi}, F_{\gamma i}\) = Inclination factors (will be discussed later).

**Notes:**
1. This equation is valid for both general and local shear failure.
2. This equation is similar to original equation for ultimate bearing capacity (Terzaghi’s equation) which derived for continuous foundation, but the shape, depth, and load inclination factors are added to this equation (Terzaghi’s equation) to be suitable for any case may exist.

**Bearing Capacity Factors:**
The angle \(\alpha = \phi\) (according Terzaghi theory in the last figure “above”) was replaced by \(\alpha = 45 + \frac{\phi}{2}\). So, the bearing capacity factor will be change.

The variations of bearing capacity factors \((N_c, N_q, N_\gamma)\) and underlying soil friction angle \(\phi\) are given in (Table 3.3, P.144).

**Shape Factors:**
\[
F_{cs} = 1 + \left(\frac{B}{L}\right) \left(\frac{N_q}{N_c}\right)
\]
\[
F_{qs} = 1 + \left(\frac{B}{L}\right) \tan \phi
\]
\[
F_{\gamma s} = 1 - 0.4 \left(\frac{B}{L}\right)
\]
Notes:
1. If the foundation is continuous or strip → \( \frac{B}{L} = 0.0 \)
2. If the foundation is circular→ \( B = L = \) diameter → \( \frac{B}{L} = 1 \)

Depth Factors:

➢ For \( \frac{D_f}{B} \leq 1 \)

1. For \( \phi = 0.0 \)
   \[
   F_{cd} = 1 + 0.4 \left( \frac{D_f}{B} \right) \\
   F_{qd} = 1 \\
   F_{yd} = 1
   \]

2. For \( \phi > 0.0 \)
   \[
   F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi} \\
   F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right) \\
   F_{yd} = 1
   \]

➢ For \( \frac{D_f}{B} > 1 \)

1. For \( \phi = 0.0 \)
   \[
   F_{cd} = 1 + 0.4 \tan^{-1} \left( \frac{D_f}{B} \right) \text{ radians} \\
   F_{qd} = 1 \\
   F_{yd} = 1
   \]

2. For \( \phi > 0.0 \)
   \[
   F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi}
   \]
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\[
F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \tan^{-1} \left( \frac{D_f}{B} \right) \text{ radians}
\]

\[
F_{yd} = 1
\]

**Important Notes:**

1. If the value of \((B)\) or \((D_f)\)is required, you should do the following:
   
   ✓ Assume \(\left( \frac{D_f}{B} \leq 1 \right)\) and calculate depth factors in term of \((B)\) or \((D_f)\).
   
   ✓ Substitute in the general equation, then calculate \((B)\) or \((D_f)\).
   
   ✓ After calculated the required value, you must check your assumption \(\left( \frac{D_f}{B} \leq 1 \right)\).
   
   ✓ If the assumption is true, the calculated value is the final required value.
   
   ✓ If the assumption is wrong, you must calculate depth factors in case of \(\left( \frac{D_f}{B} > 1 \right)\) and then calculate \((B)\) or \((D_f)\) to get the true value.

2. For both cases \(\left( \frac{D_f}{B} \leq 1 \right)\) and \(\left( \frac{D_f}{B} > 1 \right)\) if \(\phi > 0\) → calculate \(F_{qd}\) firstly, because \(F_{cd}\) depends on \(F_{qd}\).

**Inclination Factors:**

\[
F_{ci} = F_{qi} = \left( 1 - \frac{\beta^\circ}{90} \right)^2
\]

\[
F_{yi} = \left( 1 - \frac{\beta^\circ}{\phi^\circ} \right)
\]

\(\beta^\circ\) = Inclination of the load on the foundation with respect to the vertical

**Note:**

If \(\beta^\circ = \phi \rightarrow F_{yi} = 0.0\), so you **don’t need** to calculate \(F_{ys}\) and \(F_{yd}\), because the last term in Meyerhof equation will be zero.
Factor of Safety
From previous two equations (Terzaghi and Meyerhof), we calculate the value of ultimate bearing capacity \( q_u \) which the maximum value the soil can bear it (i.e. if the bearing stress from foundation exceeds the ultimate bearing capacity of the soil, shear failure in soil will be occur), so we must design a foundation for a bearing capacity less than the ultimate bearing capacity to prevent shear failure in the soil. This bearing capacity is “Allowable Bearing Capacity” and we design for it (i.e. the applied stress from foundation must not exceeds the allowable bearing capacity of soil).

\[
q_{\text{all,gross}} = \frac{q_{u,\text{gross}}}{FS} \rightarrow \text{Applied stress} \leq q_{\text{all,gross}} = \frac{q_{u,\text{gross}}}{FS}
\]

- \( q_{\text{all,gross}} \) = Gross allowable bearing capacity
- \( q_{u,\text{gross}} \) = Gross ultimate bearing capacity (Terzaghi or Meyerhof equations)
- \( FS \) = Factor of safety for bearing capacity \( \geq 3 \)

However, practicing engineers prefer to use the “net allowable bearing capacity” such that:

\[
q_{\text{all,net}} = \frac{q_{u,\text{net}}}{FS}
\]

- \( q_{u,\text{net}} \) = Net ultimate bearing capacity, and it’s the difference between the gross ultimate bearing capacity (upward as soil reaction) and the weight of the soil and foundation at the foundation level (downward), to get the net pressure from the soil that support the foundation.

\[
q_{u,\text{net}} = q_{u,\text{gross}} - \gamma_c h_c - \gamma_s h_s
\]

Since the unit weight of concrete and soil are convergent, then

\[
q_{u,\text{net}} = q_{u,\text{gross}} - q \rightarrow q_{\text{all,net}} = \frac{q_{u,\text{gross}} - q}{FS}
\]

- \( q \) = Effective stress at the level of foundation level.

If we deal with loads (\( Q \))

\[
q_{u,\text{gross}} = \frac{Q_{u,\text{gross}}}{\text{Area}} + FS \rightarrow q_{\text{all,gross}} = \frac{Q_{\text{all,gross}}}{\text{Area}}
\]
Modification of Bearing Capacity Equations for Water Table

Terzaghi and Meyerhof equations give the ultimate bearing capacity based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, the bearing capacity will decrease due to the effect of water table, so, some modification of the bearing capacity equations (Terzaghi and Meyerhof) will be necessary.

The values which will be modified are:
1. \( q \) for soil above the foundation) in the second term of equations.
2. \( \gamma \) for the underlying soil) in the third (last) term of equations.

There are three cases according to location of water table:

Case I. The water table is located so that \( 0 \leq D_1 \leq D_f \) as shown in the following figure:

\[
\begin{align*}
q &= D_1 \times \gamma + D_2 \times (\gamma_{sat} - \gamma_w) \\
\gamma &= \text{effective unit weight for soil below the foundation} \Rightarrow \gamma' = \gamma_{sat} - \gamma_w
\end{align*}
\]
Case II. The water table is located so that $0 \leq d \leq B$ as shown in the following figure:

- The factor $q$, (second term) in the bearing capacity equations will takes the following form: (For the soil above the foundation)
  
  $q = \text{effective stress at the level of the bottom of the foundation}$
  
  $\Rightarrow q = D_f \times \gamma$
  
- The factor $\gamma$, (third term) in the bearing capacity equations will takes the following form: (For the soil under the foundation)
  
  $\gamma = \text{effective unit weight for soil below the foundation at depth } d = B$
  
  I.e. calculate the effective stress for the soil below the foundation from $(d = 0 \text{ to } d = B)$, and then divide this value by depth $(d = B)$ to get the representative effective unit weight $(\bar{\gamma})$ for this depth.

  $\sigma_{0\rightarrow B}' = d \times \gamma + (B - d) \times (\gamma_{\text{sat}} - \gamma_w)$
  
  $\Rightarrow \sigma_{0\rightarrow B}' = d \times \gamma + (B - d) \times \gamma'$
  
  $\Rightarrow \frac{\sigma_{0\rightarrow B}'}{B} = \frac{d \times \gamma + B \times \gamma' - d \times \gamma'}{B}$
  
  $\Rightarrow \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B}$

Case III. The water table is located so that $d \geq B$, in this case the water table is assumed have no effect on the ultimate bearing capacity.
Eccentrically Loaded Foundation

If the load applied on the foundation is in the center of the foundation without eccentricity, the bearing capacity of the soil will be uniform at any point under the foundation (as shown in figure below) because there is no any moments on the foundation, and the general equation for stress under the foundation is:

\[
\text{Stress} = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y X}{I_y}
\]

In this case, the load is in the center of the foundation and there are no moments so,

\[
\text{Stress} = \frac{Q}{A} \text{ (uniform at any point below the foundation)}
\]

However, in several cases, as with the base of a retaining wall or neighbor footing, the loads does not exist in the center, so foundations are subjected to moments in addition to the vertical load (as shown in the below figure). In such cases, the distribution of pressure by the foundation on the soil is not uniform because there is a moment applied on the foundation and the stress
under the foundation will be calculated from the general relation:

\[ \text{Stress} = \frac{Q}{A} \pm \frac{M_x}{I_x} \pm \frac{M_y}{I_y} \]  

(in case of two way eccentricity)

But, in this section we deal with (one way eccentricity), the equation will be:

\[ \text{Stress} = \frac{Q}{A} \pm \frac{M_c}{I} \]

Since the pressure under the foundation is not uniform, there are maximum and minimum pressures (under the two edges of the foundation) and we concerned about calculating these two pressures.

**General equation for calculating maximum and minimum pressure:**

Assume the eccentricity is in direction of (B)

\[ \text{Stress} = q = \frac{Q}{A} \pm \frac{M_c}{I} \]

\[ A = B \times L \]

\[ M = Q \times e \]

\[ c = \frac{B}{2} \]  

(maximum distance from the center)

\[ I = \frac{B^3 \times L}{12} \]  

(I is about the axis that resists the moment)

Substitute in the equation, the equation will be:
\[ q = \frac{Q}{B \times L} \pm \frac{Q \times e \times B}{2 \times B^3 \times L} \rightarrow q = \frac{Q}{B \times L} \pm \frac{6eQ}{B^2 L} \rightarrow q = \frac{Q}{B \times L} \left(1 \pm \frac{6e}{B}\right) \]

\[ q = \frac{Q}{B \times L} \left(1 \pm \frac{6e}{B}\right) \text{ General Equation} \]

Now, there are three cases for calculating maximum and minimum pressures according to the values of \((e \text{ and } \frac{B}{6})\) to maintain minimum pressure always \(\geq 0\)

**Case I. (For } e < \frac{B}{6}:**

\[ q_{\text{max}} = \frac{Q}{B \times L} \left(1 + \frac{6e}{B}\right) \]
\[ q_{\text{min}} = \frac{Q}{B \times L} \left(1 - \frac{6e}{B}\right) \]

Note that when \(e < \frac{B}{6}\) the value of \(q_{\text{min}}\) will be positive (i.e. compression).

If eccentricity in \((L)\) direction:

(For } e < \frac{L}{6}):

\[ q_{\text{max}} = \frac{Q}{B \times L} \left(1 + \frac{6e}{L}\right) \]
\[ q_{\text{min}} = \frac{Q}{B \times L} \left(1 - \frac{6e}{L}\right) \]

**Case II. (For } e = \frac{B}{6}:**

\[ q_{\text{max}} = \frac{Q}{B \times L} \left(1 + \frac{6e}{B}\right) \]
\[ q_{\text{min}} = \frac{Q}{B \times L} (1 - 1) = 0.0 \]
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Note that when $e = \frac{B}{6}$ the value of $q_{\text{min}}$ will be zero (i.e. no compression and no tension) and this case is the critical case and it is accepted.

If eccentricity in (L) direction:

(For $e = \frac{L}{6}$):

$$q_{\text{max}} = \frac{Q}{B \times L} \left( 1 + \frac{6e}{L} \right)$$

$$q_{\text{min}} = \frac{Q}{B \times L} (1 - 1) = 0.0$$

**Case III. (For $e > \frac{B}{6}$):**

![Diagram](image)

As shown in the above figure (1) the value of $(q_{\text{min}})$ is negative (i.e. tension in soil), but we know that soil can’t resist any tension, thus, negative pressure must be prevented by making $(q_{\text{min}} = 0)$ at distance $(x)$ from point $(A)$ as shown in the above figure (2), and determine the new value of $(q_{\text{max}})$ by static equilibrium as following:

$$R = \text{area of triangle} \times L = \frac{1}{2} \times q_{\text{max, new}} \times X \times L \rightarrow (1)$$

$$\sum F_y = 0.0 \rightarrow R = Q \rightarrow (2)$$
\[
\sum M_{@A} = 0.0
\]
\[
\rightarrow Q \times \left( \frac{B}{2} - e \right) = R \times \frac{X}{3} \quad \text{(but from Eq. 2 \rightarrow R = Q)} \rightarrow X = 3 \left( \frac{B}{2} - e \right)
\]

Substitute by X in Eq. (1) \rightarrow
\[
R = Q = \frac{1}{2} \times q_{\text{max,new}} \times 3 \left( \frac{B}{2} - e \right) \times L \rightarrow q_{\text{max,new}} = \frac{4Q}{3L(B - 2e)}
\]

If eccentricity in (L) direction:
(For \( e > \frac{L}{2} \)):
\[
q_{\text{max,new}} = \frac{4Q}{3B(L - 2e)}
\]

**Note:**

All the above equations are derived for rectangular or square footing, but if the foundation is circular you should use the original equation for calculating the stress:
\[
q = \frac{Q}{A} \pm \frac{M_c}{I}
\]

Where
\[
A = \frac{\pi}{4} D^4 \quad \text{(D is the diameter of the circular foundation)}
\]
\[
c = \frac{D}{2}
\]
\[
I = \frac{\pi}{64} D^4
\]

And then calculate \( q_{\text{max}} \) and \( q_{\text{min}} \)
Ultimate Bearing Capacity under Eccentric Loading—One-Way Eccentricity

Effective Area Method:
As we discussed previously, if the load does not exist in the center of the foundation, or if the foundation located to moment in addition to the vertical loads, the stress distribution under the foundation is not uniform. So, to calculate the ultimate (uniform) bearing capacity under the foundation, new area should be determined to make the applied load in the center of this area and to develop uniform pressure under this new area. This new area is called Effective area. The following is how to calculate $q_u$ for this case:

1. Determine the effective dimensions of the foundation:

   Effective width $= B' = B - 2e$
   Effective Length $= L' = L$
   $B'_{used} = \min(B', L')$
   $L'_{used} = \max(B', L')$

   If the eccentricity were in the direction of (L) of the foundation:

   Effective width $= B' = B$
   Effective Length $= L' = L - 2e$
   $B'_{used} = \min(B', L')$
   $L'_{used} = \max(B', L')$

2. If we want to use terzaghi’s equation for example, for square footing:

   $q_u = 1.3cN_c + qN_q + 0.4\gamma N_Y$

   The value of B (in last term) will be $B'_{used}$ because the pressure is uniform for this value of width and the pressure does not uniform for width B. Other factors in the equation will not change.
3. If we want to use Meyerhof Equation:

\[ q_u = cN_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + 0.5B\gamma N_{\gamma} F_{\gamma s} F_{\gamma d} F_{\gamma i} \]

The value of \( B \) (in last term) will be \( B'_{\text{used}} \) to get uniform pressure on this width.

In calculating of shape factors (\( F_{cs}, F_{qs}, F_{\gamma s} \)) use \( B'_{\text{used}} \) and \( L'_{\text{used}} \) because we concerned about the shape of the footing that make the pressure uniform.

In calculating of depth factors (\( F_{cd}, F_{qd}, F_{\gamma d} \)) use the original value (\( B \)) and don’t replace it by \( B'_{\text{used}} \) due to the following two reasons:

✓ Depth factors are used to consider the depth of the foundation and thereby the depth of soil applied on the original dimensions of the foundation.
✓ In equations of depth factors, as the value of \( B \) decrease the depth factors will increase and then the value of \( q_u \) will increase, so for more safety we use the larger value of width \( B \) to decrease depth factors and thereby decrease \( q_u \) which less than \( q_u \) if we use \( B'_{\text{used}} \) (i.e. more safe).

4. If there is a water table (Case II), we need the following equation to calculate \( \gamma \) in the last term of equations (Terzaghi and Meyerhof):

\[ \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B} \]

The value of \( B \) used in this equation should be the original value (\( B \)) because we calculate the effective unit weight (\( \bar{\gamma} \)) for depth (\( B \)) below the foundation.

**Safety Consideration**

Calculate the gross ultimate load:

\[ Q_u = q_u \times \left( L'_{\text{used}} \times B'_{\text{used}} \right) \]  \( (A' = \text{effective area}) \)

The factor of safety against bearing capacity is: \( FS = \frac{Q_u}{Q_{\text{all}}} \geq 3 \)

Applied Load \( \leq Q_{\text{all}} = \frac{Q_u}{FS} \)

The factor of safety against \( q_{\text{max}} \) is: \( FS = \frac{q_u}{q_{\text{max}}} \geq 3 \)

The value of \( q_{\text{all}} \) should be equal or more than \( q_{\text{max}}: q_{\text{all}} \geq q_{\text{max}} \)

The value of \( q_{\text{min}} \) should be equal or more than zero: \( q_{\text{min}} \geq 0.0 \)
Important Notes (before solving any problem)

1. The soil above the bottom of the foundation are used only to calculate the term \( q \) in the second term of bearing capacity equations (Terzaghi and Meyerhof) and all other factors are calculated for the underlying soil.

2. Always the value of \( q \) is the effective stress at the level of the bottom of the foundation.

3. For the underlying soil, if the value of \( c = \text{cohesion} = 0.0 \) you don’t have to calculate factors in the first term in equations \( N_c \) in terzaghi’s equations and \( (N_c, F_{cs}, F_{cd}, F_{ci}) \) in Meyerhof equation.

4. For the underlying soil, if the value of \( \phi = 0.0 \) you don’t have to calculate factors in the last term in equations \( N_y \) in terzaghi’s equations and \( (N_y, F_{ys}, F_{yd}, F_{yi}) \) in Meyerhof equation.

5. If the load applied on the foundation is inclined with an angle \( \beta = \phi \) → The value of \( F_{yi} \) will be zero, so you don’t have to calculate factors in the last term of Meyerhof equation \( (N_y, F_{ys}, F_{yd}) \).

6. Always if we want to calculate the eccentricity it’s calculated as following:
   \[
   e = \frac{\text{Overall Moment}}{\text{Vertical Loads}}
   \]

7. If the foundation is square, strip or circular, you may calculate \( q_u \) from terzaghi or Meyerhof equations (should be specified in the problem).

8. But, if the foundation is rectangular, you must calculate \( q_u \) from Meyerhof general equation.

9. If the foundation width (B) is required, and there exist water table below the foundation at distance \( d \), you should assume \( d \leq B \), and calculate B, then make a check for your assumption.
Problems
1.
The square footing shown below must be designed to carry a 2400 KN load. Use Terzaghi’s bearing capacity formula and factor of safety = 3. **Determine** the foundation dimension B in the following two cases:

1. The water table is at 1m below the foundation (as shown).
2. The water table rises to the ground surface.

![Diagram](image)

\[ q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma \]

\[ q_u = q_{all} \times FS \quad (q_{all} = \frac{Q_{all}}{\text{Area}}, \quad FS = 3) \]

Applied load \( \leq Q_{all} \rightarrow Q_{all} = 2400\text{kN} \)

\[ q_{all} = \frac{Q_{all}}{\text{Area}} = \frac{2400}{B^2}, \quad FS = 3 \rightarrow q_u = \frac{3 \times 2400}{B^2} \]

\( c = 50 \text{ kN/m}^2 \)

\[ q(\text{effective stress}) = \gamma \times D_f = 17.25 \times 2 = 34.5 \text{ kN/m}^2 \]

Since the width of the foundation is not known, assume \( d \leq B \)
\[ \gamma = \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B} \]

\[ \gamma' = \gamma_{sat} - \gamma_w = 19.5 - 10 = 9.5 \text{kN/m}^3, \ d = 3 - 2 = 1 \text{m} \]

\[ \bar{\gamma} = 9.5 + \frac{1 \times (17.25 - 9.5)}{B} \rightarrow \bar{\gamma} = 9.5 + \frac{7.75}{B} \]

Assume general shear failure

**Note:**

Always we design for general shear failure (soil have a high compaction ratio) except if we can’t reach high compaction, we design for local shear (medium compaction).

For \( \phi = 32^\circ \rightarrow N_c = 44.04, \ N_q = 28.52, \ N_y = 26.87 \) (**Table 3.1**)  

Now substitute from all above factors on terzaghi equation:

\[ \frac{7200}{B^2} = 1.3 \times 50 \times 44.04 + 34.5 \times 28.52 + 0.4 \times B \times \left( 9.5 + \frac{7.75}{B} \right) \times 26.87 \]

\[ \frac{7200}{B^2} = 3923.837 + 102.106B \]

Multiply both sides by \((B^2)\) → \(102.106B^3 + 3923.837B^2 - 7200 = 0\)

\[ \rightarrow B = 1.33 \text{m } \checkmark. \]

2.

All factors remain unchanged except \(q\) and \(\gamma\):

\(q\) (effective stress) = \((19.5 - 10) \times 2 = 19 \text{ kN/m}^2\)

\(\gamma = \gamma' = 19.5 - 10 = 9.5 \text{ kN/m}^3\)

Substitute in terzaghi equation:

\[ \frac{7200}{B^2} = 1.3 \times 50 \times 44.04 + 19 \times 28.52 + 0.4 \times B \times 9.5 \times 26.87 \]

\[ \frac{7200}{B^2} = 3404.48 + 102.106B \]

Multiply both sides by \((B^2)\) → \(102.106B^3 + 3404.48B^2 - 7200 = 0\)

\[ \rightarrow B = 1.42 \text{m } \checkmark. \]

Note that as the water table elevation increase the required width \((B)\) will also increase to maintain the factor of safety (3).
2.
Determine the size of square footing to carry net allowable load of 295 KN. FS=3. Use Terzaghi equation assuming general shear failure.

Solution

\[ Q_{\text{all,net}} = 295 \text{ kN} \quad \text{and we know} \quad q_{\text{all,net}} = \frac{Q_{\text{all,net}}}{\text{Area}} \rightarrow q_{\text{all,net}} = \frac{295}{B^2} \]

Also,
\[ q_{\text{all,net}} = \frac{q_u - q}{FS} \]
q(effective stress) = \( \gamma \times D_f = 18.15 \times 1 = 18.15 \text{ kN/m}^2 \), \( FS = 3 \)

\[ \rightarrow \frac{295}{B^2} = \frac{q_u - 18.15}{3} \rightarrow q_u = \frac{885}{B^2} + 18.15 \rightarrow (1) \]

\[ q_u = 1.3cN_c + qN_q + 0.4\gamma N_\gamma \]
\[ c = 50 \text{ kN/m}^2 \]
\[ q(\text{effective stress}) = 18.15 \text{ kN/m}^2 \]
\[ \gamma = 20 \text{ kN/m}^3 \text{ (for underlying soil)} \]
For \( \phi = 25^\circ \rightarrow N_c = 25.13, \ N_q = 12.72, \ N_\gamma = 8.34 \text{ (Table 3.1)} \)

Substitute from all above factor in Terzaghi equation:
\[ q_u = 1.3 \times 50 \times 25.13 + 18.15 \times 12.72 + 0.4 \times B \times 20 \times 8.34 \]
\[ \rightarrow q_u = 1864.318 + 66.72B \]

Substitute from Eq. (1):
\[ \frac{885}{B^2} + 18.15 = 1864.318 + 66.72B \]
Multiply both side by \(B^2\):
\[ 66.72 B^3 + 1846.168B^2 - 885 = 0 \]
\[ \rightarrow B = 0.68 \text{ m } \checkmark. \]
3.
For the square footing (2.5m x 2.5m) shown in the figure below, determine the allowable resisting moment (M), if the allowable load \( P = 800 \text{ KN} \) and F.S = 3. (Using Meyerhof Equation).

![Figure with dimensions and geometry](image)

\[ \gamma_s = 20 \text{ kN/m}^3 \]

**Solution**

\[ M = Q \times e = 800e \]

\[ q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_{\gamma}F_{\gamma}F_{\gamma d}F_{\gamma i} \]

The first term in the equation will be zero because \( c = 0 \), so the equation will be:

\[ q_u = qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_{\gamma}F_{\gamma}F_{\gamma d}F_{\gamma i} \]

\( q \) (effective stress) = \( \gamma \times D_f = 16.8 \times 1.5 = 25.2 \text{ kN/m}^2 \)

**Calculating the new area that maintains \( q_u \) uniform:**

\( B' = B - 2e \rightarrow B' = 2.5 - 2e \), \( L' = 2.5 \)

\( B'_{used} = \min(B',L') = 2.5 - 2e \), \( L'_{used} = 2.5 \text{ m} \)

\[ q_u = q_{all} \times \text{FS} \quad (q_{all} = \frac{Q_{all}}{A'} \rightarrow A' = B'_{used} \times L'_{used} \ , \ \text{FS} = 3) \]

Applied load \( \leq Q_{all} \rightarrow Q_{all} = 800\text{kN} \)

\[ q_{all} = \frac{800}{(2.5 - 2e) \times 2.5} = \frac{320}{2.5 - 2e} \quad \rightarrow \quad q_u = 3 \times \frac{320}{2.5 - 2e} = \frac{960}{2.5 - 2e} \]
d = 1m ≤ B = 2.5m → water table will effect on $q_u$ →

$$
\gamma = \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B} \quad \text{(Use B not $B'_\text{used}$ as we explained previously)}
$$

$\gamma' = \gamma_{\text{sat}} - \gamma_w = 20 - 10 = 10 \text{ kN/m}^3$, $d = 1m$, $\gamma = 16.8 \text{ kN/m}^3$ →

$$
\bar{\gamma} = 10 + \frac{1 \times (16.8 - 10)}{2.5} = 12.72 \text{ kN/m}^3
$$

**Bearing Capacity Factors:**

For $\phi = 35^\circ \rightarrow N_c = 57.75$, $N_q = 41.44$, $N_\gamma = 45.41 \quad (\text{Table 3.3})$

**Shape Factors:**

As we explained previously, use $B'_\text{used}$ and $L'_\text{used}$

$$
F_{cs} = 1 + \left( \frac{B'_\text{used}}{L'_\text{used}} \right) \left( \frac{Nq}{N_c} \right) \quad \text{does not required (because } c = 0.0)$$

$$
F_{qs} = 1 + \left( \frac{B'_\text{used}}{L'_\text{used}} \right) \tan \phi = 1 + \left( \frac{2.5 - 2e}{2.5} \right) \times \tan 35 = 1.7 - 0.56 \, e
$$

$$
F_{ys} = 1 - 0.4 \left( \frac{B'_\text{used}}{L'_\text{used}} \right) = 1 - 0.4 \times \left( \frac{2.5 - 2e}{2.5} \right) = 0.6 + 0.32 \, e
$$

**Depth Factors:**

As we explained previously, use B not $B'_\text{used}$

$$
\frac{D_f}{B} = \frac{1.5}{2.5} = 0.6 < 1 \quad \text{and } \phi = 35 > 0.0 \rightarrow \rightarrow
$$

$$
F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi} \quad \text{does not required (because } c = 0.0)
$$

$$
F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right)
$$

$$
= 1 + 2 \tan 35 \times (1 - \sin 35)^2 \times 0.6 = 1.152
$$

$F_{yd} = 1$

**Inclination Factors:**

The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhof equation:

$$
\frac{960}{2.5 - 2e} = 25.2 \times 41.44 \times (1.7 - 0.56 \, e) \times 1.152
$$

$$
+ 0.5 \times (2.5 - 2e) \times 12.72 \times 45.41 \times (0.6 + 0.32 \, e)
$$
\frac{960}{2.5 - 2e} = 2478.345 - 789.2e - 184.832e^2

Multiply both sides by \((2.5 - 2e)\):

\(960 = 6195.825 - 4956.66e - 1973e + 1578044e^2 - 462e^2 + 369.6e^3\)

\(\rightarrow 369.6e^3 + 1116.44e^2 - 6929.7e + 5235.825 = 0.0\)

Solve for \(e\)\(\rightarrow e = 2.36\) or \(e = 0.94\) or \(e = -6.33\)

Now, the value of \((e)\) must be less than \(\frac{B}{2}\) and must be positive value

\(B = \frac{2.5}{2} = 1.25 < 2.36\) \(\rightarrow\) reject the value of \(e = 2.36\) and negative value

\(\rightarrow e = 0.94\) m

\(M = Q \times e = 800 \times 0.94 = 752\) kN.m \(\checkmark\).

4.

For the soil profile is given below, determine the allowable bearing capacity of the isolated rectangular footing (2m x 2.3m) that subjected to a vertical load (425 kN) and moment (85 kN.m), \(FS=3\).

\(\phi = 20^\circ\)

\(C = 35\) kN/m\(^2\)

\(\gamma_d = 16\) kN/m\(^3\)

\(\phi = 25^\circ\)

\(C = 0\)

\(\gamma_s = 19\) kN/m\(^3\)
Solution

\[ q_u = q_{all} \times FS \rightarrow q_{all} = \frac{q_u}{FS} \rightarrow q_{all} = \frac{q_u}{3} \]

\[ q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_YF_{ys}F_{yd}F_{yi} \]

Note that the value of \((c)\) for the soil under the foundation equal zero, so the first term in the equation will be terminated (because we calculate the bearing capacity for soil below the foundation) and the equation will be:

\[ q_u = qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_YF_{ys}F_{yd}F_{yi} \]

q (effective stress) = \(\gamma \times D_f = 16 \times 1.5 = 24\) kN/m²

Calculating the new area that maintains \(q_u\) uniform:

Note that the eccentricity in the direction of \((L=2.3)\)

\[ e = \frac{M}{Q} = \frac{85}{425} = 0.2m \]

\[ B' = B = 2m \rightarrow L' = L - 2e = 2.3 - 2 \times 0.2 = 1.9m \]

\[ B'_{used} = \min(B',L') = 1.9m \rightarrow L'_{used} = 2m \]

Effective Area \((A') = 1.9 \times 2 = 3.8\) m²

Water table is at the bottom of the foundation \(\rightarrow \gamma = \gamma' = \gamma_s - \gamma_w\)

\(\rightarrow \gamma = \gamma' = 19 - 10 = 9\) kN/m³

Bearing Capacity Factors:

For \(\phi = 25^\circ\) \(\rightarrow N_c = 20.72, N_q = 10.66, N_Y = 10.88\) (Table 3.3)

Shape Factors:

As we explained previously, use \(B'_{used}\) and \(L'_{used}\)

\[ F_{cs} = 1 + \left( \frac{B'_{used}}{L'_{used}} \right) \left( \frac{N_q}{N_c} \right) \] does not required (because \(c = 0.0\))

\[ F_{qs} = 1 + \left( \frac{B'_{used}}{L'_{used}} \right) \tan\phi = 1 + \left( \frac{1.9}{2} \right) \times \tan25 = 1.443 \]

\[ F_{ys} = 1 - 0.4 \left( \frac{B'_{used}}{L'_{used}} \right) = 1 - 0.4 \times \left( \frac{1.9}{2} \right) = 0.62 \]

Depth Factors:

As we explained previously, use \(B\) not \(B'_{used}\)
$D_f = \frac{1.5}{2} = 0.75 < 1$ and $\phi = 25 > 0.0 \rightarrow$

$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi}$ does not required (because $c = 0.0$)

$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right) = 1 + 2 \tan 25 \times (1 - \sin 25)^2 \times 0.75 = 1.233$

$F_{\gamma d} = 1$

**Inclination Factors:**
The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhofer equation:

$q_u = 24 \times 10.66 \times 1.443 \times 1.233 + 0.5 \times 1.9 \times 9 \times 10.88 \times 0.62 \times 1$

$\rightarrow q_u = 512.87 \text{kN/m}^2$

$q_{all} = \frac{q_u}{3} = \frac{512.87}{3} = 170.95 \text{kN/m}^2 \checkmark$

If the required is net allowable bearing capacity $q_{all,net}$:

$\frac{q_{all,net}}{FS} = \frac{512.87 - 24}{3} = 162.95 \text{kN/m}^2 \checkmark$. 
5.
An eccentrically loaded rectangular foundation (6ft x 8ft) shown below. Use factor of safety of 3 and if \( e = 0.5 \text{ft} \), determine the allowable load that the foundation could carry. (The factor of safety is based on the maximum stress along the base of the footing).

\[
\begin{align*}
\gamma_d &= 110 \text{pcf} \\
\gamma_s &= 122.4 \text{pcf} \\
\phi &= 15^\circ \\
C &= 800 \text{psf} \\
C &= 800 \text{lbs/ft}^2
\end{align*}
\]

Solution

Note that the factor of safety is for \( q_{\text{max}} \rightarrow \text{FS} = \frac{q_u}{q_{\text{max}}} \geq 3 \), but in the previous problems the factor of safety is for bearing capacity \( \text{FS} = \frac{q_u}{q_{\text{all}}} \geq 3 \)
That means the factor of safety is always for bearing capacity, except if it’s specified like this problem (FS for \( q_{\text{max}} \)).

\[
q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma\gamma_sF_{yd}F_{yi}
\]

\[ c = 800 \text{ lbs/ft}^2 \]

\[ q(\text{effective stress}) = 110 \times 3 + (122.4 - 62.4) \times 4 = 570 \text{ lb/ft}^2 \]

**Calculating the new area that maintains \( q_u \) uniform:**

Note that the eccentricity in the direction of (B=6)

\[ e = 0.5 \text{ft} \]

\[ B' = B - 2e = 6 - 2 \times 0.5 = 5 \text{ft} \]

\[ L' = L = 8 \text{ft} \]
B'\text{used} = \min(B', L') = 5\text{ft} \quad L'_\text{used} = 8\text{ft}

Effective Area (A') = 5 \times 8 = 40 \text{ ft}^2

Water table is above the bottom of the foundation → γ = γ' = γ_s - γ_w
→ γ = γ' = 122.4 - 62.4 = 60 \text{ lb/ft}^3

**Bearing Capacity Factors:**
For \( \phi = 15^\circ \) → \( N_c = 10.98, N_q = 3.94, N_\gamma = 2.65 \) (Table 3.3)

**Shape Factors:**
As we explained previously, use \( B'\text{used} \) and \( L'_\text{used} \)
\[
F_{cs} = 1 + \left( \frac{B'_{\text{used}}}{L'_{\text{used}}} \right) \left( \frac{N_q}{N_c} \right) = 1 + \left( \frac{5}{8} \right) \left( \frac{3.94}{10.98} \right) = 1.224
\]
\[
F_{qs} = 1 + \left( \frac{B'_{\text{used}}}{L'_{\text{used}}} \right) \tan\phi = 1 + \left( \frac{5}{8} \right) \times \tan15 = 1.167
\]
\[
F_{ys} = 1 - 0.4 \left( \frac{B'_{\text{used}}}{L'_{\text{used}}} \right) = 1 - 0.4 \times \left( \frac{5}{8} \right) = 0.75
\]

**Depth Factors:**
As we explained previously, use \( B \) not \( B'\text{used} \)
\[
D_f \quad B = \frac{7}{6} = 1.16 > 1 \quad \text{and} \quad \phi = 15 > 0.0 \rightarrow \rightarrow
\]
\[
F_{qd} = 1 + 2 \tan\phi \,(1 - \sin\phi)^2 \, \tan^{-1} \left( \frac{D_f}{B} \right)
\]
\[
\tan^{-1} \left( \frac{D_f}{B} \right) = \tan^{-1} \left( \frac{7}{6} \right) = 0.859 \text{ radians}
\]
→ \( F_{qd} = 1 + 2 \tan(15) \times (1 - \sin15)^2 \times 0.859 = 1.252 \)
\[
F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi} = 1.252 - \frac{1 - 1.252}{10.98 \times \tan(15)} = 1.337
\]
\[
F_{\gamma d} = 1
\]

**Inclination Factors:**
The load on the foundation is not inclined, so all inclination factors are (1).
Now substitute from all above factors in Meyerhof equation:

\[ q_u = 800 \times 10.98 \times 1.224 \times 1.337 + 570 \times 3.94 \times 1.167 \times 1.252 \\
+ 0.5 \times 5 \times 60 \times 2.65 \times 0.75 \times 1 \]

\[ \rightarrow q_u = 17954.34 \text{ lb/ft}^2 \]

Now, to calculate \( q_{\text{max}} \) we firstly should check the value of \( e = 0.5 \text{ft} \)

\[ \frac{B}{6} = \frac{6}{6} = 1 \text{ft} \rightarrow e = 0.5 < \frac{B}{6} = 1 \rightarrow \]

\[ q_{\text{max}} = \frac{Q}{B \times L} \left( 1 + \frac{6e}{B} \right) \]

\[ q_{\text{max}} = \frac{Q}{6 \times 8} \left( 1 + \frac{6 \times 0.5}{6} \right) = 0.03125Q \]

\[ \text{FS} = \frac{q_u}{q_{\text{max}}} = 3 \rightarrow q_u = 3q_{\text{max}} \]

\[ \rightarrow 17954.34 = 3 \times 0.03125Q \rightarrow Q = 191512.96 \text{ lb} = 191.5 \text{ Kips} \checkmark. \]

6.
For the rectangular foundation (2m x 3m) shown below:
a) Compute the net allowable bearing capacity (FS=3).
b) If the water table is lowered by 2m. What effect on bearing capacity would occur due to the water lowering?

\[ \gamma_d = 18 \text{ kN/m}^3 \]
\[ \phi = 25^\circ \]
\[ C = 0.0 \]
\[ \gamma_s = 21 \text{ kN/m}^3 \]
Solution

**Important Note:**
The load on the foundation is considered inclined, when this load is applied directly on the foundation, however if the load does not applied directly on the foundation (like this problem), this load is not considered inclined.

The analysis of the inclined load (700 KN) on the column will be as shown in figure below:

The inclined load on the column will be divided into two components (vertical and horizontal):
Vertical component = 700 × sin60 = 606.2 KN
Horizontal component = 700 × cos60 = 350 kN
The horizontal component will exert moment on the foundation in the direction shown in figure above:
\[ M = 350 \times 1.5 = 525 \text{ kN.m} \]
\[ e = \frac{\text{Overall moment}}{\text{Vertical Load}} = \frac{525}{606.2} = 0.866 \text{ m} \]

\( q_{\text{all,net}} = \frac{q_u - q}{FS} \)
\( q_u = cN_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + 0.5B\gamma N_{\gamma} F_{\gamma} F_{yd} F_{\gamma i} \)
Note that the value of \( c \) for the soil under the foundation equal zero, so the first term in the equation will be terminated (because we calculate the bearing capacity for soil below the foundation) and the equation will be:
\( q_u = qN_q F_{qs} F_{qd} F_{qi} + 0.5B\gamma N_{\gamma} F_{\gamma} F_{yd} F_{\gamma i} \)
q(effective stress) = 18 × 0.5 + (21 − 10) × 1 = 20 kN/m²
Calculating the new area that maintains $q_u$ uniform:
Note that the eccentricity in the direction of $(L=3)$
e = 0.866m
$B' = B = 2 \text{ m} \rightarrow L' = L - 2e \rightarrow L' = 3 - 2 \times 0.866 = 1.268 \text{ m}$
$B'_{\text{used}} = \min(B',L') = 1.268 \text{ m} \text{, } L'_{\text{used}} = 2 \text{ m}$
Effective Area $(A') = 1.268 \times 2 = 2.536 \text{ m}^2$
Water table is above the bottom of the foundation $\rightarrow \gamma = \gamma' = \gamma_s - \gamma_w$ $\rightarrow \gamma = \gamma' = 21 - 10 = 11 \text{ kN/m}^3$

**Bearing Capacity Factors:**
For $\phi = 25^\circ \rightarrow N_c = 20.72, N_q = 10.66, N_\gamma = 10.88$ (Table 3.3)

**Shape Factors:**
As we explained previously, use $B'_{\text{used}}$ and $L'_{\text{used}}$

$F_{cs} = 1 + \left( \frac{B'_{\text{used}}}{L'_{\text{used}}} \right) \left( \frac{N_q}{N_c} \right) \text{ does not required (because } c = 0.0)$

$F_{qs} = 1 + \left( \frac{B'_{\text{used}}}{L'_{\text{used}}} \right) \tan \phi = 1 + \left( \frac{1.268}{2} \right) \times \tan 25 = 1.296$

$F_{\gamma s} = 1 - 0.4 \left( \frac{B'_{\text{used}}}{L'_{\text{used}}} \right) = 1 - 0.4 \times \left( \frac{1.268}{2} \right) = 0.746$

**Depth Factors:**
As we explained previously, use $B$ not $B'_{\text{used}}$

$D_f \frac{B}{2} = 1.5 < 1 \text{ and } \phi = 25 > 0.0 \rightarrow \rightarrow$

$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi} \text{ does not required (because } c = 0.0)$

$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right)$

$= 1 + 2 \tan 25 \times (1 - \sin 25)^2 \times 0.75 = 1.233$

$F_{\gamma d} = 1$

**Inclination Factors:**
The load on the foundation is not inclined, so all inclination factors are (1).
Now substitute from all above factors in Meyerhof equation:

\[ q_u = 20 \times 10.66 \times 1.296 \times 1.233 + 0.5 \times 1.268 \times 11 \times 10.88 \times 0.746 \]

\[ \rightarrow q_u = 397.29 \text{ kN/m}^2 \]

\[ q_{all,net} = \frac{q_u - q}{FS} = \frac{397.29 - 20}{3} = 125.76 \text{ kN/m}^2 \checkmark. \]

b)
This case is shown in the below figure:

All factors remain unchanged except \( q \) and \( \gamma \):
\[ q(\text{effective stress}) = \gamma \times D_f = 18 \times 1.5 = 27 \text{ kN/m}^2 \]

\( d = 1 \text{ m} \leq B = 2 \text{ m} \rightarrow \) water table will effect on \( q_u \)
\[ \gamma = \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B} \] (Use \( B \) not \( B'_{\text{used}} \) as we explained previously)

\[ \gamma' = \gamma_{\text{sat}} - \gamma_w = 21 - 10 = 11 \text{ kN/m}^3, \quad d = 1 \text{ m}, \quad \gamma = 18 \text{ kN/m}^3 \rightarrow \]

\[ \bar{\gamma} = 11 + \frac{1 \times (18 - 11)}{2} = 14.5 \text{ kN/m}^3 \]

Substitute in Meyerhof equation:
\[ q_u = 27 \times 10.66 \times 1.296 \times 1.233 + 0.5 \times 1.268 \times 14.5 \times 10.88 \times 0.746 \]

\[ \rightarrow q_u = 534.54 \text{ kN/m}^2 \]

The effect of water lowering is increase \( q_u \) by \( 534.5 - 397.3 = 137.2 \text{ kN/m}^2 \checkmark. \)
7. For the rectangular footing (2.5 m x 3 m) shown below, if $e = 0.35 \text{ m}$ and $q_{\text{max}} = 410 \text{ kN/m}^2$. Calculate the factor of safety against bearing capacity, and determine whether the design is good or not.

![Diagram of a rectangular footing](image)

- $\phi = 30^\circ$
- $C = 30 \text{ kN/m}^2$
- $\gamma_s = 21 \text{ kN/m}^3$

Solution

Note that the inclined load is applied directly on the foundation, so it is an inclined load with angle ($\beta = 90 - 60 = 30^\circ$ with vertical).

$$FS = \frac{Q_u}{Q_{\text{all}}} \quad , \quad Q_u = q_u \times A' \quad , \quad Q_{\text{all}} = ??$$

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma\gamma_NF_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Since $\beta = \phi = 30^\circ$, the inclination factor $F_{\gamma i}$ will equal zero, so the last term in equation will be terminated and the equation will be:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi}$$

- $c = 30 \text{ kN/m}^2$
- $q(\text{effective stress}) = 15 \times 0.5 + (21 - 10) \times 1 = 18.5 \text{ kN/m}^2$
Calculating the new area that maintains $q_u$ uniform:
Note that the eccentricity in the direction of (L=3)
$e = 0.35m$
$B' = B = 2.5 m \rightarrow L' = L - 2e \rightarrow L' = 3 - 2 \times 0.35 = 2.3m$
$B'_{used} = \min(B', L') = 2.3 m \rightarrow L'_{used} = 2.5 m$

Effective Area ($A'$) = $2.3 \times 2.5 = 5.75 \text{ m}^2$

Bearing Capacity Factors:
For $\phi = 30^\circ \rightarrow N_c = 30.14$, $N_q = 18.4$, $N_\gamma = 22.4$ (Table 3.3)

Shape Factors:
As we explained previously, use $B'_{used}$ and $L'_{used}$

$$F_{cs} = 1 + \left( \frac{B'_{used}}{L'_{used}} \right) \left( \frac{N_q}{N_c} \right) = 1 + \left( \frac{2.3}{2.5} \right) \left( \frac{18.4}{30.14} \right) = 1.56$$

$$F_{qs} = 1 + \left( \frac{B'_{used}}{L'_{used}} \right) \tan\phi = 1 + \left( \frac{2.3}{2.5} \right) \times \tan30 = 1.53$$

$$F_{ys} = 1 - 0.4 \left( \frac{B'_{used}}{L'_{used}} \right) \text{ does not required (because } \beta = \phi = 30^\circ \text{)}$$

Depth Factors:
As we explained previously, use $B$ not $B'_{used}$

$$D_f = \frac{1.5}{2.5} = 0.6 < 1 \text{ and } \phi = 30 > 0.0 \rightarrow$$

$$F_{qd} = 1 + 2 \tan\phi (1 - \sin\phi)^2 \left( \frac{D_f}{B} \right)$$

$$F_{qd} = 1 + 2 \tan(30) (1 - \sin30)^2 \times 0.6 = 1.173$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi} = 1.173 - \frac{1 - 1.173}{30.14 \times \tan(30)} = 1.183$$

$$F_{yd} = 1$$

Inclination Factors:

$$F_{ci} = F_{qi} = \left( 1 - \frac{\beta}{90} \right)^2 = \left( 1 - \frac{30}{90} \right)^2 = 0.444$$

$$F_{yi} = 0.0$$
Now substitute from all above factors in Meyerhof equation:

\[ \begin{align*}
    q_u &= 30 \times 30.14 \times 1.56 \times 1.183 \times 0.444 \\
         & \quad + 18.5 \times 18.4 \times 1.53 \times 1.173 \times 0.444 \\
    \rightarrow q_u &= 1012.14 \text{ kN/m}^2
\end{align*} \]

\[ Q_u = q_u \times A' = 1012.14 \times 5.75 = 5819.8 \text{ KN} \]

\[ e = 0.35 \text{ m } , \quad \frac{L}{6} = 0.5 \text{ m } \rightarrow e = 0.35 < \frac{L}{6} = 0.5 \rightarrow \rightarrow \]

We used term (L) because eccentricity in L direction

\[ \begin{align*}
    q_{\text{max}} &= 410 = \frac{Q_{\text{all}}}{B \times L} \left( 1 + \frac{6e}{L} \right) \rightarrow 410 = \frac{Q_{\text{all}}}{2.5 \times 3} \times \left( 1 + \frac{6 \times 0.35}{3} \right) \rightarrow \rightarrow \\
    Q_{\text{all}} &= 1808.3 \text{ KN} \\
    \text{FS} &= \frac{Q_u}{Q_{\text{all}}} = \frac{5819.8}{1808.3} = 3.22 \checkmark.
\end{align*} \]

Since the factor of safety is larger than 3, the design is good \( \checkmark \).
8.
A square footing 2.5m x 2.5m is shown in the figure below. If the maximum pressure on the foundation should not exceed the allowable bearing capacity. Using factor of safety (FS=3), find the maximum horizontal force that the foundation can carry if the water table is 1m below the foundation. (Use Terzaghi equation)

\[
\gamma_d = 17 \text{kN/m}^3 \\
\phi = 30^\circ \\
C = 50 \text{kN/m}^2 \\
\gamma_{sat} = 19.5 \text{kN/m}^3
\]

**Solution**
The following figure explains the analysis of the given loads:

\[
e = \frac{\text{Overall moment}}{\text{Vertical Load}} = \frac{165 + 1.5H}{300} = 0.55 + 0.005H \rightarrow (1)
\]
**Ultimate Bearing Capacity of Shallow Foundations**

\[ q_{\text{max}} \leq q_{\text{all}} \text{(given)} \rightarrow q_{\text{all}} = q_{\text{max}} \text{ (To get maximum value of } H) \]

\[ \rightarrow \text{FS} = \frac{q_u}{q_{\text{all}}} \rightarrow 3 = \frac{q_u}{q_{\text{all}}} \rightarrow q_u = 3q_{\text{all}} \text{ so, } q_u = 3q_{\text{max}} \]

\[ q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma \]

\[ c = 50 \text{ kN/m}^2 \]

\[ q(\text{effective stress}) = 17 \times 1.5 = 25.5 \text{ kN/m}^2 \]

**Calculating the new area that maintains } q_u \text{ uniform:**

\[ B' = B - 2e = 2.5 - 2e \rightarrow , \ L' = B = 2.5 \]

\[ B'_{\text{used}} = \min(B', L') = 2.5 - 2e , \ L'_{\text{used}} = 2.5m \]

Effective Area \( (A') = (2.5 - 2e) \times 2.5 = 6.25 - 5e \)

\[ d = 1m \leq B = 2.5m \rightarrow \text{water table will effect on } q_u \rightarrow \]

\[ \gamma = \overline{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B} \text{ (Use } B \text{ not } B'_{\text{used}} \text{ as we explained previously)} \]

\[ \gamma' = \gamma_{\text{sat}} - \gamma_{w} = 19.5 - 10 = 9.5 \text{ kN/m}^3 \text{ , } d = 1m \text{ , } \gamma = 17 \text{ kN/m}^3 \rightarrow \]

\[ \overline{\gamma} = 9.5 + \frac{1 \times (17 - 9.5)}{2.5} = 12.5 \text{ kN/m}^3 \]

**Bearing Capacity Factors:**

For \( \phi = 30^\circ \rightarrow N_c = 37.16, \ N_q = 22.46, \ N_\gamma = 19.13 \text{ (Table 3.1)} \)

Substitute from all above factors in Terzaghi equation:

\[ q_u = 1.3 \times 50 \times 37.16 + 25.5 \times 22.46 + 0.4 \times (2.5 - 2e) \times 12.5 \times 19.13 \]

\[ q_u = 3227.25 - 191.3e \]

**Calculating of } q_{\text{max}}:**

\[ B = \frac{2.5}{6} = 0.416 , \ e = 0.55 + 0.005H \]

(Note that the first term of \( e = 0.55 > \frac{B}{6} = 0.416 \rightarrow e > \frac{B}{6} \rightarrow \))

Use the modified equation for \( q_{\text{max}}:**

\[ q_{\text{max,modified}} = \frac{4Q}{3L(B - 2e)} = \frac{4 \times 300}{3 \times 2.5 \times (2.5 - 2e)} = \frac{160}{2.5 - 2e} \]

\[ q_u = 3q_{\text{max}} \rightarrow 3227.25 - 191.3e = 3 \times \frac{160}{2.5 - 2e} \]
Multiply both side by \((2.5 - 2e)\):

\[382.6e^2 - 6932.75e + 7588.125 = 0.0 \rightarrow e = 1.17 \text{ m}\]

Substitute in Eq.(1):

\[1.17 = 0.55 + 0.005H \rightarrow H = 124 \text{ kN} \checkmark\]