Sanitary Engineering Discussion
Problems

1.
A pipe 300 mm internal diameter, and 50 m long. Its upper and lower manholes are collected as shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Ground Level</th>
<th>Invert Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream Manhole</td>
<td>15.3 m</td>
<td>14 m</td>
</tr>
<tr>
<td>Downstream Manhole</td>
<td>14.7 m</td>
<td>13.5 m</td>
</tr>
</tbody>
</table>

If the water consumption is 150 l/c/day and manning coefficient is 0.013. Estimate how much population from Gaza such pipe can serve?

**Hint:** Use common design criteria

**Solution**

**Note:** Common design criteria means, check for maximum and minimum flows and velocities and use $\frac{d}{D} = 0.67$

$Q_P = Q_{avg(ww)} \times P.f$

The following equation for peak factor is given:

$P.f = 1 + \frac{14}{4 + \sqrt{P_{thousands}}}$

$Q_P = \frac{0.8 \times 150 \times P}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{0.001P}}\right) \rightarrow\rightarrow\text{Eq. (1)}$

Note that if we calculate $Q_P$, we can calculate $P$

Common design criteria $\rightarrow \frac{d}{D} = 0.67 \rightarrow \frac{Q_P}{Q_f} = 0.785$ (from charts)

Now, to calculate $Q_P$, we must calculate $Q_f$

Manning equation:

$Q_f = \frac{0.312}{n} \times \frac{8}{D^3} \times S^{\frac{1}{2}}$

$n = 0.013$, $D = 0.3 m$, but $S = ??$

We calculate $S$ from the given elevations and the given length of the pipe, but! We calculate the slope from **invert levels** not ground levels because the actual slope of the pipe is from invert level:

$S = \frac{14 - 13.5}{50} = 0.01 > 0.005 \rightarrow S = 0.01$

$\rightarrow Q_f = \frac{0.312}{0.013} \times (0.3)^{\frac{8}{3}} \times (0.01)^{\frac{1}{2}} = 0.096 \text{ m}^3/\text{s}$
\[ Q_P = \frac{Q_P}{Q_f} \times Q_f = 0.785 \times 0.096 = 0.0754 \text{ m}^3/\text{s} \rightarrow \text{Sub. in Eq. (1)} \]

\[ 0.0754 = \frac{0.8 \times 150 \times P}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{0.001P}}\right) \]

\[ \rightarrow P = 20,560 \text{ inhabitant } \checkmark. \]

2.

Two cities are to be served by a wastewater gravity sewer system. At 2010 city (1) has a population of 60,000 inhabitant and increase with arithmetic growth of 1000 inhabitant/year. City (2) has a population of 50,000 inhabitants with geometric growth rate of 2.5%. The area of each city is 300,000 m\(^2\) and each city has a saturated population density of 0.4 inhabitant/m\(^2\). Design pipe A and pipe C using the common design criteria with manning coefficient of 0.013, average water consumption of 130 l/c/day, and design year is 2050.

Commercial pipe diameters: 8”, 10”, 12”, ….., 40” (2 steps).
Solution

**Design of pipe A:**
The wastewater in pipe A are from city one inly:
Calculate the population of city one:
Arithmetic growth equation is: \( P_f = P_o + kt \)
\( t = 40 \text{ yr} \), \( k = 1000 \text{ inhabitant/yr} \)
\( \rightarrow P_f = 60,000 + 1000 \times 40 = 100,000 \text{ inhabitant} \)

Saturation population for city one:
\( P_{s\text{saturation}} = \text{saturated density} \times \text{city area} \)
\( \rightarrow P_{s\text{saturation}} = 0.4 \frac{\text{inhabitant}}{\text{m}^2} \times 300,000\text{m}^2 = 120,000 \text{ inhabitant} \)

The final population of city one is:
\( P = \text{Min.}(100,000; 120,000) = 100,000 \text{ inhabitant} \)

Now we calculate \( Q_P \):
\( Q_P = Q_{\text{avg}(w\text{w})} \times P.f \)
If you are given more than one equation for peak factor you must calculate \( P.f \) from each equation and then take the maximum value, but in most cases you given only the following equation:
\( P.f = 1 + \frac{4}{4 + \sqrt{P_{\text{thousands}}}} \)
\( Q_P = Q_{\text{avg}(w\text{w})} \times P.f \)
\( Q_P = \frac{0.8 \times 130 \times 100,000}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{100}}\right) = 0.2407 \text{m}^3/\text{s} \)

For common design criteria \( \frac{d}{D} = 0.67 \rightarrow \frac{Q_P}{Q_f} = 0.785 \) (from charts)
\( Q_f = \frac{Q_P}{Q_f} = \frac{0.2407}{0.785} = 0.306 \text{ m}^3/\text{s} \)

Manning equation:
\( Q_f = \frac{0.312}{n} \times D^{\frac{8}{3}} \times S^{\frac{1}{2}} \)
From the given invert levels and length of pipe A, we calculate the slope:
\( S = \frac{40 - 30}{2000} = 0.005 = \text{min. slope} \rightarrow S = 0.005 \)
0.306 = \frac{0.312}{0.013} \times (D)^{\frac{8}{3}} \times (0.005)^{\frac{1}{2}} \rightarrow D = 0.526 m

\frac{0.526 \times 100}{2.54} = 20.7 \text{ inch but we given } 22 \text{ inch } \rightarrow D = 22 \text{ inch}

D_{\text{new, meter}} = \frac{22 \times 2.54}{100} = 0.558 m

Q_{f, \text{new}} = \frac{0.312}{0.013} \times (0.558)^{\frac{8}{3}} \times (0.005)^{\frac{1}{2}} = 0.358 \text{ m}^3/s

\left( \frac{Q_p}{Q_f \text{ new}} \right) = \frac{0.2407}{0.358} = 0.672 \rightarrow \frac{d}{D} = 0.6 \text{ (from charts)}

\frac{d}{D} = 0.6 \rightarrow \frac{V_p}{V_f} = 1.08 \text{ (from charts)}

V_f = \frac{1}{n} \times s^{\frac{1}{2}} \times \left( \frac{D}{4} \right)^{\frac{2}{3}} \text{ (Manning equation)}

V_f = \frac{1}{0.013} \times (0.005)^{\frac{1}{2}} \times \left( \frac{0.558}{4} \right)^{\frac{2}{3}} = 1.46 m/s

V_p = \frac{V_p}{V_f} \times V_f = 1.08 \times 1.46 = 1.5768 m/s

0.6 < 1.5678 < 3 \rightarrow \text{Ok ✓}

**Design of pipe C:**

Note that (from the given invert levels for all pipes), city one and city two are served by pipe C, so the total population served by pipe C is the population of city one and city two.

Calculate the population of city one:

P = 100,000 inhabitant \text{ (as calculated above)}

Calculate the population of city two:

Geometric growth equation is: \( P_f = P_o(1 + k)^t \)

\( t = 40 \text{ yr} \), \( k = 2.5\% \)

\( \rightarrow P_f = 50,000(1 + 0.025)^{40} = 134,253 \text{ inhabitant} \)

Saturation population for city one:

\( P_{\text{saturation}} = \text{saturated density} \times \text{city area} \)

\( \rightarrow P_{\text{saturation}} = 0.4 \frac{\text{inhabitant}}{\text{m}^2} \times 300,000\text{m}^2 = 120,000 \text{ inhabitant} \)
The final population of city one is:
\[ P_f = \text{Min} \, (134,253 ; 120,000) = 120,000 \text{ inhabitant} \]

So, total population served by pipe C is:
\[ P = 100,000 + 120,000 = 220,000 \text{ inhabitant} \]

Now we calculate \( Q_p \):
\[
Q_p = Q_{\text{avg}(ww)} \times P, f
\]
\[
Q_p = \frac{0.8 \times 130 \times 220,000}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{220}}\right) = 0.461 \text{ m}^3/\text{s}
\]

For common design criteria \( \frac{d}{D} = 0.67 \rightarrow \frac{Q_p}{Q_f} = 0.785 \) (from charts)
\[
Q_f = \frac{Q_p}{\frac{Q_p}{Q_f}} = \frac{0.461}{0.785} = 0.584 \text{ m}^3/\text{s}
\]

Manning equation:
\[
Q_f = \frac{0.312}{n} \times D^8 \times S^\frac{1}{2}
\]

From the given invert levels and length of pipe C, we calculate the slope:
\[
S = \frac{30 - 20}{2000} = 0.005 = \text{min. slope} \rightarrow S = 0.005
\]

\[
0.584 = \frac{0.312}{0.013} \times (D)^8 \times (0.005)^{\frac{1}{2}} \rightarrow D = 0.67 \text{ m}
\]

\[
D = \frac{0.67 \times 100}{2.54} = 26.38 \text{ inch} \text{ but we given 28 inch} \rightarrow D = 28 \text{ inch}
\]

\[
D_{\text{new, meter}} = \frac{28 \times 2.54}{100} = 0.7112 \text{ m}
\]

\[
Q_{f,\text{new}} = \frac{0.312}{0.013} \times (0.7112)^8 \times (0.005)^{\frac{1}{2}} = 0.684 \text{ m}^3/\text{s}
\]

\[
\left(\frac{Q_p}{Q_f}\right)_\text{new} = \frac{0.461}{0.684} = 0.674 \rightarrow \frac{d}{D} = 0.61 \text{ (from charts)}
\]

\[
\frac{d}{D} = 0.6 \rightarrow \frac{V_p}{V_f} = 1.07 \text{ (from charts)}
\]

\[
V_f = \frac{1}{n} \times S^\frac{1}{2} \times \left(\frac{D}{4}\right)^{\frac{2}{3}} \text{ (Manning equation)}
\]

\[
V_f = \frac{1}{0.013} \times (0.005)^{\frac{1}{2}} \times \left(\frac{0.7112}{4}\right)^{\frac{2}{3}} = 1.72 \text{ m/s}
\]
\[ V_p = \frac{V_P}{V_f} \times V_f = 1.07 \times 1.72 = 1.84 \text{ m/s} \]

\[ 0.6 < 1.84 < 3 \rightarrow \text{Ok } \checkmark \]

**Important Note:** If the velocity is smaller than 0.6, you should provide your answer by the following comment: “Not Ok, to increase velocity we increase the slope of the pipe or increase the diameter of the pipe”.

However, if the velocity is larger than 3, you should provide your answer by the following comment: “Not Ok, to decrease velocity we decrease the slope of the pipe or decrease the diameter of the pipe”.

3.

In a waste water gravity system that is served two cities with population of 50,000 inhabitant for each city at 2010. There are two alternatives to be used to serve these cities as shown in the figure below. Knowing that the construction cost of 1m of the pipe is shown in the table (according pipe diameter). Choose the optimum alternative using the common design criteria. Manning coefficient is 0.015 and average water consumption is 150 l/c/day. Growth rate is 2% and design year is 2050.

![Alternative (1) and Alternative (2) diagrams](image-url)
For alternative (1):
We find the diameters if all pipes and then we calculate the total cost for all pipes of this alternative.
Note that, the diameter of pipe A and pipe B will be the same, since all data about these two pipes are the same.

**Design of pipe A:**
Pipe A carrying wastewater coming from city one.
Calculate the population of city one:
Geometric growth equation is: \( P_f = P_o(1 + k)^t \)
\( t = 40 \text{ yr} \), \( k = 2\% \)
\( \rightarrow P_f = 50,000(1 + 0.02)^{40} = 110,402 \text{ inhabitant} \)
Now we calculate \( Q_P \):
\[
Q_P = Q_{avg(ww)} \times P.f
\]
\[
Q_P = \frac{0.8 \times 150 \times 110,402}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{110.402}}\right) = 0.301 \text{m}^3/\text{s}
\]

**Note:**
In any problem you may given “the waste water production = 120 l/c/d”, here when we calculate \( Q_P \) without multiplying it by 0.8, because we directly given the production of waste water.
For common design criteria \( \frac{d}{D} = 0.67 \rightarrow \frac{Q_P}{Q_f} = 0.785 \) (from charts)
\[
Q_f = \frac{Q_P}{Q_f} = \frac{0.301}{0.785} = 0.381 \text{ m}^3/\text{s}
\]
Manning equation:

\[ Q_f = \frac{0.312}{n} \times D^\frac{8}{3} \times S^\frac{1}{2} \]

From the given invert levels and length of pipe A, we calculate the slope:

\[ S = \frac{40 - 30}{1500} = 0.0067 > \text{min. slope} \rightarrow S = 0.0067 \]

\[ 0.381 = \frac{0.312}{0.015} \times (D)^\frac{8}{3} \times (0.0067)^\frac{1}{2} \rightarrow D = 0.57 \text{ m} \]

\[ D = \frac{0.57 \times 100}{2.54} = 22.4 \text{ inch but given 24 inch} \rightarrow D = 24 \text{ inch} \]

\[ D_{\text{new, meter}} = \frac{24 \times 2.54}{100} = 0.6096 \text{ m} \]

\[ Q_{f,\text{new}} = \frac{0.312}{0.015} \times (0.6096)^\frac{8}{3} \times (0.0067)^\frac{1}{2} = 0.453 \text{ m}^3/\text{s} \]

\[ \left( \frac{Q_p}{Q_f} \right)_{\text{new}} = \frac{0.301}{0.453} = 0.66 \rightarrow \frac{d}{D} = 0.6 \text{ (from charts)} \]

\[ \frac{d}{D} = 0.6 \rightarrow \frac{V_p}{V_f} = 1.08 \text{ (from charts)} \]

\[ V_f = \frac{1}{n} \times S^\frac{1}{2} \times \left( \frac{D}{4} \right)^\frac{2}{3} \] (Manning equation)

\[ V_f = \frac{1}{0.013} \times (0.0067)^\frac{1}{2} \times \left( \frac{0.6096}{4} \right)^\frac{2}{3} = 1.55 \text{ m/s} \]

\[ V_p = \frac{V_p}{V_f} \times V_f = 1.08 \times 1.55 = 1.67 \text{ m/s} \]

\[ 0.6 < 1.67 < 3 \rightarrow \text{Ok} \checkmark \]

So, the diameter of pipes A and B is 24 inch.

**Design of pipe C:**

Note that (from the given invert levels for all pipes), city one and city two are served by pipe C, so the total population served by pipe C is the population of city one and city two.

City one and city two have the same population as calculated above:

So, total population served by pipe C is:

\[ P = 110,402 + 110,402 = 220,804 \text{ inhabitant} \]

Now we calculate \( Q_p \):
\[ Q_P = Q_{avg(ww)} \times P.f \]
\[ Q_P = \frac{0.8 \times 150 \times 220,804}{1000 \times 24 \times 3600} \times \left( 1 + \frac{14}{4 + \sqrt{220.804}} \right) = 0.534 \text{m}^3/\text{s} \]

For common design criteria: \( \frac{d}{D} = 0.67 \rightarrow \frac{Q_P}{Q_f} = 0.785 \) (from charts)

\[ Q_f = \frac{Q_P}{\frac{Q_P}{Q_f}} = 0.534 \div 0.785 = 0.677 \text{ m}^3/\text{s} \]

Manning equation:
\[ Q_f = \frac{0.312}{n} \times \left( \frac{D}{S} \right)^{\frac{8}{2}} \]

From the given invert levels and length of pipe C, we calculate the slope:
\[ S = \frac{30 - 20}{2000} = 0.0067 = \text{min. slope} \rightarrow S = 0.0067 \]
\[ 0.677 = \frac{0.312}{0.015} \times (D)^{\frac{8}{3}} \times (0.0067)^{\frac{1}{2}} \rightarrow D = 0.708 \text{ m} \]
\[ D = \frac{0.708 \times 100}{2.54} = 27.88 \text{ inch} \text{ but we given 28 inch} \rightarrow D = 28 \text{ inch} \]
\[ D_{new, meter} = \frac{28 \times 2.54}{100} = 0.711 \text{ m} \]

Now, continue and check velocity by yourself 😊

Now, we calculate the total cost for alternative one, from the given table:
\[ \text{Cost} = 2 \times 1500 \text{m} \times 275 \frac{\text{m}}{\text{m}} + 1500 \text{m} \times 350 \frac{\text{m}}{\text{m}} = 1.35 \text{ M} \$

**For alternative (2):**
Design pipes A’ and B’ and then calculate the cost from the table, but we note that pipes A’ and B’ have the same data and hence have the same diameter.
Design by yourself 😊😊
After design process, the diameter of pipes A’ and B’ is 22 inch.

Now, we calculate the total cost for alternative two, from the given table:
\[ \text{Cost} = 2 \times 2500 \text{m} \times 235 \frac{\text{m}}{\text{m}} = 1.175 \text{ M} \$

The optimum alternative is the alternative which having minimum cost, so we choose **alternative (2). ✓**
4. A 16” diameter pipe is to be used in a wastewater gravity system. For this pipe calculate the following:

a) The maximum flow that can be carried by this pipe.
b) The maximum velocity that can be occur in the pipe.
c) The population that can served by this pipe to the common design criteria.
d) The flow depth and velocity when the pipe is carrying the minimum flow.

**Given:**

- \( n = 0.013 \), average water consumption = 130 l/c/day, \( S = 1\% \), minimum flow factor = 0.2 \( P^{1/6} \)

**Solution**

Firstly, from the given data we calculate the full flow and the full velocity:

\[
Q_f = \frac{0.312}{n} \times D^\frac{8}{3} \times S^\frac{1}{2}, \quad V_f = \frac{1}{n} \times S^\frac{1}{2} \times \left(\frac{D}{4}\right)^\frac{2}{3}
\]

\( n = 0.013 \), \( D = \frac{16 \times 2.54}{100} = 0.4064 \text{ m} \), \( S = 0.01 \)

\[
\rightarrow Q_f = \frac{0.312}{0.013} \times (0.4064)^\frac{8}{3} \times (0.01)^\frac{1}{2} = 0.2175 \text{ m}^3/\text{s}
\]

\[
\rightarrow V_f = \frac{1}{0.013} \times (0.01)^\frac{1}{2} \times \left(\frac{0.4064}{4}\right)^\frac{2}{3} = 1.685 \text{ m/s}
\]

a) The maximum flow in the pipe occur when \( \frac{d}{D} = 0.95 \)

\[ @ \frac{d}{D} = 0.95 \rightarrow \frac{Q_P}{Q_f} = 1.09 \text{ (from charts)} \]

\[
Q_{P,max} = \frac{Q_P}{Q_f} \times Q_f = 1.09 \times 0.2175 = 0.237 \text{ m}^3/\text{s} \checkmark
\]

b) The maximum velocity in the pipe occur when \( \frac{d}{D} = 0.81 \)

\[ @ \frac{d}{D} = 0.81 \rightarrow \frac{V_P}{V_f} = 1.15 \text{ (from charts)} \]

\[
V_{P,max} = \frac{V_P}{V_f} \times V_f = 1.15 \times 1.675 = 1.926 \text{ m/s} \checkmark
\]

c) Common design criteria at \( \frac{d}{D} = 0.67 \)

\[ @ \frac{d}{D} = 0.67 \rightarrow \frac{Q_P}{Q_f} = 0.785 \]

\[
\rightarrow Q_P = \frac{Q_P}{Q_f} \times Q_f = 0.785 \times 0.2175 = 0.1716 \text{ m}^3/\text{s}
\]
\[ Q_P = 0.1716 = \frac{0.8 \times 130 \times P}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{0.001P}}\right) \]

\[ \rightarrow P = 66,192 \text{ inhabitant} \checkmark. \]

d) \[ Q_{\text{Min}} = Q_{\text{avg(ww)}} \times MF \]

\[ Q_{\text{avg(ww)}} = \frac{0.8 \times 130 \times P}{1000 \times 24 \times 3600} \] (we use \(P\) calculated above)

\[ Q_{\text{avg(ww)}} = \frac{0.8 \times 130 \times 66,192}{1000 \times 24 \times 3600} = 0.0796 \text{ m}^3/\text{s} \]

MF = minimum flow factor = 0.2P^{1/6}_{\text{thousands}}

\[ \rightarrow MF = 0.2 \times 66.192^{1/6} = 0.402 \]

\[ \rightarrow Q_{\text{Min}} = 0.0796 \times 0.402 = 0.032 \text{ m}^3/\text{s} \]

\[ \rightarrow Q_{\text{partial}} = Q_{\text{Min}} = 0.032 \text{ m}^3/\text{s} \]

\[ \frac{Q_P}{Q_f} = \frac{0.032}{0.2175} = 0.147 \rightarrow \frac{d}{D} = 0.25 \text{ (from charts)} \]

\[ \frac{d}{D} = 0.25, D = 0.4064 \rightarrow d = \text{flow depth} = 0.25 \times 0.4064 = 0.1016 \text{m} \checkmark. \]

\[ @ \frac{d}{D} = 0.25 \rightarrow \frac{V_P}{V_f} = 0.7 \text{ (from charts)} \]

\[ V_P = \frac{V_P}{V_f} \times V_f = 0.7 \times 1.685 = 1.17 \text{ m/s} \checkmark. \]

5.
A sewage line is to be designed to serve a city with population at 2000 of 80,000 inhabitant. The average water consumption is 100 l/c/d and increases every year by 5 l/c/d. The infiltration rate is 40 m$^3$/hr and the growth rate is 3.5% and the slope of the pipe is 4%. Design the pipe to carry the flow year 2025. Manning coefficient is 0.015.

Solution

Calculate the population at 2025:

\[ P_f = 80,000(1 + 0.035)^{25} = 189,060 \text{ inhabitant} \]

Calculate the daily consumption at 2025:

\[ \text{Consumption} = 100 + 5 \times 25 = 225 \text{ l/c/d} \]
Calculate the partial design flow:

\[ Q_{\text{design}} = Q_{\text{peak}} + Q_{\text{infiltration}} + Q_{\text{inflow}} \]

\[ Q_{\text{Peak}} = Q_{\text{avg}}(ww) \times P \cdot f \]

\[ Q_{\text{Peak}} = \frac{0.8 \times 225 \times 189,060}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{189.060}}\right) = 0.7045 \text{ m}^3/\text{s} \]

\[ Q_{\text{infiltration}} = 40 \text{ m}^3/\text{hr} = \frac{40}{60 \times 60} = 0.011 \text{ m}^3/\text{s} \]

\[ Q_{\text{inflow}} = 0.0 \]

\[ Q_{\text{design}} = 0.7045 + 0.011 + 0.0 = 0.7156 \text{ m}^3/\text{s} \]

\[ Q_{\text{design}} = Q_P = 0.7156 \]

For common design criteria → \( \frac{d}{D} = 0.67 \) → \( \frac{Q_P}{Q_f} = 0.785 \) (from charts)

\[ Q_f = \frac{Q_P}{Q_f} = \frac{0.7156}{0.785} = 0.9111 \text{ m}^3/\text{s} \]

Manning equation:

\[ Q_f = \frac{0.312}{n} \times D^\frac{8}{3} \times S^\frac{1}{2} \]

\[ S = 0.04 > S_{\text{max}} \text{ but we use the given slope (0.04) since it required} \]

\[ 0.911 = \frac{0.312}{0.015} \times (D)^{\frac{8}{3}} \times (0.04)^{\frac{1}{2}} \rightarrow D = 0.57 \text{ m} \]

\[ D = \frac{0.57 \times 100}{2.54} = 22.4 \text{ inch but we given 24 inch} \rightarrow D = 24 \text{ inch} \]

\[ D_{\text{new, meter}} = \frac{24 \times 2.54}{100} = 0.6096 \text{ m} \]

\[ Q_{f,\text{new}} = \frac{0.312}{0.015} \times (0.6096)^{\frac{8}{3}} \times (0.04)^{\frac{1}{2}} = 1.11 \text{ m}^3/\text{s} \]

\[ \left(\frac{Q_P}{Q_f}\right)_{\text{new}} = \frac{0.7156}{1.11} = 0.64 \rightarrow \frac{d}{D} = 0.59 \text{ (from charts)} \]

\[ @ \frac{d}{D} = 0.59 \rightarrow \frac{V_P}{V_f} = 1.06 \text{ (from charts)} \]

\[ V_f = \frac{1}{n} \times S^\frac{1}{2} \times \left(\frac{D}{4}\right)^\frac{2}{3} \text{ (Manning equation)} \]

\[ V_f = \frac{1}{0.015} \times (0.04)^{\frac{1}{2}} \times \left(\frac{0.6096}{4}\right)^\frac{2}{3} = 3.8 \text{ m/s} \]
\[ V_P = \frac{V_p}{V_f} \times V_f = 1.06 \times 3.8 = 4.032 \text{ m/s} \]

4.032 < 3 \rightarrow \text{Not Ok} \] Since we use \( S = 0.04 > S_{\text{max}}, \rightarrow \) to decrease velocity, we decrease \( S \) (preferable) or \( D \checkmark \).

**Note:**

In this problem, if minimum flow is required, we calculate it as following:

\[ Q_{\text{min}} = Q_{\text{avg(ww)}} \times MF + Q_{\text{infiltration}} + 0.0 \]

**Important Notes about Inverted Siphon:**

✓ It is preferable to consists of three pipes.
✓ The flow in each of three pipes is full flow, so can use manning equation directly.
✓ All pipes have the same length and the same slope.
✓ The velocity in each of three pipes must be greater than \( 0.9 \text{ m/s} \) which is the minimum velocity in inverted siphon.
✓ The slope of these pipes is calculated as following:

\[ S = \frac{\text{Length of pipe}}{\text{IL}_{\text{upper manhole}} - \text{IL}_{\text{lower manhole}}} \]
✓ The flow in each of three pipes is calculated as following:

\[ Q_1 = Q_{\text{min}} = Q_{\text{avg(ww)}} \times MF \]
\[ Q_2 = Q_{\text{avg(ww)}} - Q_{\text{min}} \]
\[ Q_3 = Q_{\text{design}} - Q_{\text{avg(ww)}} \]

**Note that:** \( Q_1 + Q_2 + Q_3 = Q_{\text{design}} \) (Which is the design flow)

\[ Q_{\text{design}} = Q_{\text{peak}} + Q_{\text{infiltration}} + Q_{\text{inflow}} \]

In most cases: \( Q_{\text{infiltration}} = Q_{\text{inflow}} = 0.0 \)

\[ \rightarrow Q_{\text{design}} = Q_{\text{peak}} = Q_{\text{avg(ww)}} \times P.f \]
6. An inverted siphon composed of three pipes is to be used to convey wastewater from a city of total population 225000 inhabitants at side of a river to the other side. The water consumption is 120 L/c/d. The invert level of the upstream manhole is + 98 m above the sea level. The length of each of the three pipes is 300 m. The peak factor = 2.5 and minimum flow factor = 0.40. Consider Manning coefficient = 0.013

a) Find the average flow in m³/s.
b) Find the flow in each of the three pipes of the inverted siphon in m³/s.
c) Find the diameter of the minimum flow pipe considering minimum velocity when flowing full is 0.90 m/s.
d) Determine the invert level of the minimum flow pipe at the downstream manhole.

Solution

a) 
\[ Q_{avg(ww)} = \frac{0.8 \times 120 \times 225000}{1000 \times 24 \times 3600} = 0.25 \text{ m}^3/\text{s} \] ✓

b) 
\[ Q_1 = Q_{min} = Q_{avg(ww)} \times MF \quad (MF = 0.4 \text{ as given}) \]
\[ \rightarrow Q_1 = Q_1 = Q_{min} = 0.25 \times 0.4 = 0.1 \text{ m}^3/\text{s} \] ✓

\[ Q_2 = Q_{avg(ww)} - Q_{min} = 0.25 - 0.1 = 0.15 \text{ m}^3/\text{s} \] ✓

\[ Q_3 = Q_{design} - Q_{avg(ww)} = Q_{avg(ww)} \times P.f - Q_{avg(ww)} \]
\[ (P.f = 2.5 \text{ as given}) \rightarrow Q_3 = 0.25 \times 2.5 - 0.25 = 0.375 \text{ m}^3/\text{s} \] ✓

c) 
\[ Q = Q_{min} = 0.1, \quad V = 0.9 \]
There are two unknowns: Slope and diameter.

\[ Q_f = \frac{0.312}{n} \times \frac{8}{D^3} \times \frac{1}{S^2} \rightarrow 0.1 = \frac{0.312}{0.013} \times \frac{8}{D^3} \times \frac{1}{S^2} \rightarrow \text{ Eq. (1)} \]

\[ V_f = \frac{1}{n} \times \frac{1}{S^2} \times \left(\frac{D}{4}\right)^{\frac{3}{2}} \rightarrow 0.9 = \frac{1}{0.013} \times \frac{1}{S^2} \times \left(\frac{D}{4}\right)^{\frac{3}{2}} \rightarrow \text{ Eq. (2)} \]

By solving the two equations:
\[ D = 0.375 \text{ m} \] ✓ and \[ S = 0.00324 \]
Or, \( Q = AV \) (since the flow is full)
\[ 0.1 = 0.9A \Rightarrow A = 0.11 \text{ m}^2 = \frac{\pi}{4} \times D^2 \Rightarrow D = 0.375 \text{ m} \checkmark. \]

d) \[ S = \frac{IL_{\text{upper manhole}} - IL_{\text{lower manhole}}}{\text{Length of pipe}} \rightarrow 0.00324 = \frac{98 - IL_{\text{lower manhole}}}{300} \]

\[ IL_{\text{lower manhole}} = 97.028 \text{ m} \checkmark. \]

**Note:**
The value of slope (0.00324) is less than minimum slope, but we considered it because we asked to achieve \( Q_{\text{min}} \) and \( V_{\text{min}} \) only.

7.
A city with population of 50,000 inhabitant is to be served with a 3 parallel pipes inverted siphon. The length of each pipe is 600m and the water consumption is 150 l/c/d.

**Design** the inverted siphon knowing that the upper invert level is +100m MSL. (The lower invert level is not known).

Commercial Pipes: 8”, 10”….40” (2step) with \( n = 0.015 \), \( MF = 0.2 P^{1/6} \)

**Solution**

The slope of each three pipes is unknown.

We can assume the diameter of the minimum flow pipe is the minimum given diameter (8”).

Let \( D_1 = D_{\text{min}} = 8” = 0.2032 \text{ m} \)

The slope of this pipe must satisfy minimum flow and minimum velocity.

\[ Q_1 = Q_{\text{min}} = Q_{\text{avg(ww)}} \times MF \]
\[ MF = 0.2 \times 50^{1/6} = 0.384 \]
\[ Q_{\text{avg(ww)}} = \frac{0.8 \times 150 \times 50,000}{1000 \times 24 \times 3600} = 0.069 \text{ m}^3/\text{s} \]

\[ Q_1 = Q_{\text{min}} = 0.384 \times 0.069 = 0.0265 \text{ m}^3/\text{s} \]

\[ V_{\text{min}} = 0.9 \text{ m/s} \]

\[ Q_f = \frac{0.312}{n} \times \frac{8}{n} D^3 \times S^2 \rightarrow 0.0265 = \frac{0.312}{0.015} \times (0.2032)^{3/2} \times S^2 \]
\[ \rightarrow S = 0.0082 \] (this slope satisfy \( Q_{\text{min}} \))

\[ V_f = \frac{1}{n} \times S^{3/2} \times \left( \frac{D}{4} \right)^{2/3} \rightarrow 0.9 = \frac{1}{0.015} \times S^{3/2} \times \left( \frac{0.2032}{4} \right)^{2/3} \]
So, the minimum slope that can satisfy $V_{\text{min}}$ is 0.0096 so we take $S = 0.0096$. But since 0.0096 > 0.0082 the value of $Q_{\text{min}}$ will change:

$$Q_{\text{min,new}} = \frac{0.312}{0.015} \times (0.2032)^{\frac{8}{3}} \times (0.0096)^{\frac{1}{2}} = 0.029 \text{ m}^3/\text{s}$$

So, $D_1 = 8'' ✓$ and $S = 0.0096$ (for all pipes)

**Design of pipe 2:**

$$Q_2 = Q_{\text{avg(ww)}} - Q_{\text{min}} = 0.069 - 0.029 = 0.04 \text{ m}^3/\text{s}$$

$$0.04 = \frac{0.312}{0.015} \times (D_2)^{\frac{8}{3}} \times (0.0096)^{\frac{1}{2}} \rightarrow D_2 = 0.229 \text{ m} = 9''$$

But we take $D_2 = 10'' ✓$ (given pipes) = 0.254m

So, the value of $Q_2$ will change

$$Q_2,\text{new} = \frac{0.312}{0.015} \times (0.254)^{\frac{8}{3}} \times (0.0096)^{\frac{1}{2}} = 0.0527 \text{ m}^3/\text{s}$$

$$Q_2 = Q_{\text{avg(ww)}} - Q_{\text{min}} \rightarrow Q_{\text{avg(ww),new}} = 0.0527 + 0.029 = 0.0817$$

Check velocity:

$$V_f = \frac{1}{0.015} \times (0.0096)^{\frac{1}{2}} \times \left(\frac{0.254}{4}\right)^{\frac{2}{3}} = 1.04 \text{ m/s} > 0.9 \text{ Ok}$$

**Design of pipe 3:**

$$Q_3 = Q_{\text{peak}} - Q_{\text{avg(ww),new}}$$

$$Q_{\text{peak}} = Q_{\text{avg(ww),original}} \times P.f$$

$$\rightarrow Q_{\text{peak}} = 0.069 \times \left(1 + \frac{14}{4 + \sqrt{50}}\right) = 0.157 \text{ m}^3/\text{s}$$

$$\rightarrow Q_3 = 0.157 - 0.0817 = 0.0753 \text{ m}^3/\text{s}$$

$$0.0753 = \frac{0.312}{0.015} \times (D_3)^{\frac{8}{3}} \times (0.0096)^{\frac{1}{2}} \rightarrow D_3 = 0.29 \text{ m} = 11.4''$$

But we take $D_2 = 12'' ✓$ (given pipes) = 0.3048m

Check velocity:

$$V_f = \frac{1}{0.015} \times (0.0096)^{\frac{1}{2}} \times \left(\frac{0.3048}{4}\right)^{\frac{2}{3}} = 1.17 \text{ m/s} > 0.9 \text{ Ok}$$
8.
In a wastewater collection system that is served a city with a population of 62,000 inhabitants, the water will be transferred from the side A to existing manhole located at side B as shown in the figure below.

a. Determine the system you will design from A to B.
b. Design the system from A to B.

Knowing that:-
Average water Consumption = 150 l/c/d
Commercial Pipes:-
8”, 10”, 12”, 14”, 16”, 18”. Manning coefficient = 0.013
20, 24, 28, 32….. 4” step …… Manning coefficient= 0.015.

Hint (May you don’t need to make siphon from A to B).

Solution
Note that we can’t make inverted siphon from A to B since the given distance from A to C (100) is horizontal and the siphon pipes don’t be horizontal. So we can make inverted siphon from C to B and make a pipe (with partial flow) from A to C.

So, the system is: Partial flow pipe from A to C and Inverted Siphon composed of three pipes from C to B ✓.

Design the pipe from A to C:
The pipe will be designed for the peak flow:

\[ Q_P = Q_{avg(ww)} \times P.f \]

\[ Q_{avg(ww)} = \frac{0.8 \times 150 \times 62000}{1000 \times 24 \times 3600} = 0.0861 \text{ m}^3/\text{s} \]
P. f = 1 + \frac{14}{4 + \sqrt{62}} = 2.18
→ Q_p = 0.0861 \times 2.18 = 0.187 \text{ m}^3/\text{s}

For common design criteria → \frac{d}{D} = 0.67 → \frac{Q_p}{Q_f} = 0.785 (from charts)

\[ Q_f = \frac{Q_p}{\frac{d}{D}} = \frac{0.187}{0.67} = 0.283 \text{ m}^3/\text{s} \]

Manning equation:

\[ Q_f = \frac{0.312}{n} \times D^{\frac{8}{3}} \times S^{\frac{1}{2}} \]

Assume the diameter of pipe is less than or equal 18”, so \( n = 0.013 \).

Note that, the slope from A to C is not given, so we assume it 0.01 (between max. and min. slopes).

\[ 0.238 = \frac{0.312}{0.013} \times (D)^{\frac{8}{3}} \times (0.01)^{\frac{1}{2}} \rightarrow D = 0.42 \text{ m} \]

\[ D = \frac{0.42 \times 100}{2.54} = 16.55 \text{ inch but we given 18 inch } \rightarrow D = 18" \text{ (n = ok)} \]

\[ D_{\text{new, meter}} = \frac{18 \times 2.54}{100} = 0.4572 \text{ m} \]

\[ Q_{f,\text{new}} = \frac{0.312}{0.013} \times (0.4572)^{\frac{8}{3}} \times (0.01)^{\frac{1}{2}} = 0.297 \text{ m}^3/\text{s} \]

\[ \left( \frac{Q_p}{Q_f} \right)_{\text{new}} = \frac{0.187}{0.297} = 0.629 \rightarrow \frac{d}{D} = 0.58 \text{ (from charts)} \]

@ \frac{d}{D} = 0.58 \rightarrow \frac{V_p}{V_f} = 1.02 \text{ (from charts)}

\[ V_f = \frac{1}{n} \times S^{\frac{1}{2}} \times \left( \frac{D}{4} \right)^{\frac{2}{3}} \text{ (Manning equation)} \]

\[ V_f = \frac{1}{0.013} \times (0.01)^{\frac{1}{2}} \times \left( \frac{0.4572}{4} \right)^{\frac{2}{3}} = 1.811 \text{ m/s} \]

\[ V_p = \frac{V_p}{V_f} \times V_f = 1.02 \times 1.811 = 1.85 \text{ m/s} \]

0.6 < 1.85 < 3 → Ok✓.
**Design the inverted siphon from C to B:**

Firstly we calculate the slope of the inverted siphon pipes:

Calculate the ground level at C:

\[
S = \frac{GL_A - GL_C}{100} = 0.01 = \frac{99 - GL_C}{100} \rightarrow GL_C = 98m
\]

To calculate the slope of the inverted siphon, we should know the invert level at point C (since IL at point B is known).

To calculate the IL at point C we assume the diameter of the pipe and hence the minimum cover of this pipe.

Assume \(D = 18"\) (max. diameter) = 0.4572 m → Cover = 1.3 m

To make invert level for each of three pipes the same, we take the invert level of the maximum pipe diameter.

\[
IL_C = GL_C - Cover - Diameter \text{ (with neglect pipe thickness)}
\]

\[
IL_C = 98 - 1.3 - 0.4572 = 96.242m
\]

So, the slope of the inverted siphon is:

\[
S = \frac{96.242 - 90}{480} = 0.013 > 0.005 \text{ Ok.}
\]

**Design of pipe 1:**

\[
Q_1 = Q_{min} = Q_{avg(ww)} \times MF
\]

\[
MF = 0.2 \times 62^{1/6} = 0.397
\]

\[
\rightarrow Q_1 = Q_{min} = 0.0817 \times 0.397 = 0.0342 \text{ m}^3/\text{s}
\]

\[
0.0342 = \frac{0.312}{0.013} \times (D_1)^{\frac{8}{3}} \times (0.013)^{\frac{1}{2}} \rightarrow D_1 = 0.193m = 7.6"\]

But we take \(D_1 = 8"\) ✓ (given pipes) = 0.2032m ✓

So, the value of \(Q_1\) will change

\[
Q_{1, new} = \frac{0.312}{0.013} \times (0.2032)^{\frac{8}{3}} \times (0.013)^{\frac{1}{2}} = 0.039 \text{ m}^3/\text{s}
\]

Check velocity:

\[
V_f = \frac{1}{0.013} \times (0.013)^{\frac{1}{2}} \times \left(\frac{0.2032}{4}\right)^{\frac{2}{3}} = 1.2 \text{ m/s} > 0.9 \text{ Ok}
\]
**Design of pipe 2:**

\[ Q_2 = Q_{\text{avg(ww)}} - Q_{\text{min,new}} = 0.0817 - 0.039 = 0.0426 \text{ m}^3/\text{s} \]

\[ 0.0426 = \frac{0.312}{0.013} \times (D_2)^{\frac{8}{3}} \times (0.013)^{\frac{1}{2}} \rightarrow D_2 = 0.21 \text{ m} = 8.26" \]

But we take \( D_2 = 10" \) ✓ (given pipes) = 0.254m ✓

So, the value of \( Q_2 \) will change

\[ Q_{2,\text{new}} = \frac{0.312}{0.013} \times (0.254)^{\frac{8}{3}} \times (0.013)^{\frac{1}{2}} = 0.0708 \text{ m}^3/\text{s} \]

\[ Q_2 = Q_{\text{avg(ww)}} - Q_{\text{min}} \rightarrow Q_{\text{avg(ww),new}} = 0.0708 + 0.039 = 0.1098 \]

Check velocity:

\[ V_f = \frac{1}{0.013} \times (0.013)^{\frac{1}{2}} \times \left(\frac{0.254}{4}\right)^{\frac{2}{3}} = 1.4 \text{ m/s} > 0.9 \text{ Ok} \]

**Design of pipe 3:**

\[ Q_3 = Q_{\text{peak}} - Q_{\text{avg(ww),new}} \]

\[ Q_{\text{peak}} = Q_{\text{avg(ww),original}} \times P.f \]

\[ \rightarrow Q_{\text{peak}} = 0.0817 \times 2.18 = 0.178 \text{ m}^3/\text{s} \]

\[ \rightarrow Q_3 = 0.178 - 0.1098 = 0.0682 \text{ m}^3/\text{s} \]

\[ 0.0682 = \frac{0.312}{0.013} \times (D_3)^{\frac{8}{3}} \times (0.013)^{\frac{1}{2}} \rightarrow D_3 = 0.25 \text{ m} = 9.86" \]

But we take \( D_3 = 10" \) ✓ (given pipes) = 0.254m ✓

Note that, the diameter is the same with pipe 2, and pipe 3 should have a diameter larger than pipe 2, so we take \( D_3 = 12" = 0.3048 \text{ m} \)

Check velocity:

\[ V_f = \frac{1}{0.013} \times (0.013)^{\frac{1}{2}} \times \left(\frac{0.3048}{4}\right)^{\frac{2}{3}} = 1.57 \text{ m/s} > 0.9 \text{ Ok} \]

Now, we can calculate the actual IL at point C and the actual slope of the inverted siphon.

\[ \text{IL}_C = 98 - 1.2 - 0.3048 = 96.4952 \text{ m} \]

\[ S = \frac{96.4952 - 90}{480} = 0.0135 \cong 0.013. \]
1.

A catchment area of 1.5 km$^2$. The land use of the area is as shown in the following table:

<table>
<thead>
<tr>
<th>Type</th>
<th>Percent Area</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open area</td>
<td>10</td>
<td>0.3</td>
</tr>
<tr>
<td>Roofs</td>
<td>40</td>
<td>0.6</td>
</tr>
<tr>
<td>Roads</td>
<td>25</td>
<td>0.9</td>
</tr>
<tr>
<td>Pavements</td>
<td>25</td>
<td>0.9</td>
</tr>
</tbody>
</table>

**Determine** the commercial diameter of the outlet pipe taking slope of 0.5$\%$ and storm intensity of 15 mm/hr.

**Solution**

Calculation of discharge served by the pipe: $Q = CIA$

To calculate the runoff coefficient for the whole area, we should calculate the composite runoff coefficient by weighted average method:

$$C = \frac{0.3 \times (0.1 \times 1.5) + 0.6 \times (0.4 \times 1.5) + 0.9 \times (0.25 \times 1.5) \times 2}{1.5}$$

$\rightarrow C = 0.72$

$I = 15 \text{ mm/hr} = \frac{15 \times 10^{-3}}{60 \times 60} = 4.167 \times 10^{-6} \text{ m/sec}$

$A = 1.5 \text{ Km}^2 = 1.5 \times 10^6 \text{ m}^2$

$\rightarrow Q = Q_f = CIA = 0.72 \times 4.167 \times 10^{-6} \times 1.5 \times 10^6 = 4.5 \text{ m}^3/\text{sec}$

Now, since the piped serve the whole area, we use Manning’s equation to calculate the diameter of this pipe:

$$Q = \frac{0.312}{n} \times S^{0.5} \times D^{8/3}$$

$4.5 = \frac{0.312}{0.013} \times 0.005^{0.5} \times D^{8/3} \rightarrow D = 1.44 \text{ m} \text{ (Use } D = 1.5 \text{ m)}$

$Q_{f,\text{new}} = \frac{0.312}{0.013} \times 0.005^{0.5} \times 1.5^{8/3} = 5 \text{ m}^3/\text{sec}$

$Q_p = 4.5 \frac{\text{m}^3}{\text{sec}} \rightarrow \frac{Q_p}{Q_{f,\text{new}}} = \frac{4}{5} = 0.9 \rightarrow \frac{d}{D} = 0.72 \rightarrow \frac{V_p}{V_f} = 1.09$

$V_f = \frac{1}{0.013} \times 0.005^{0.5} \times \left(\frac{1.5}{4}\right)^{\frac{2}{3}} = 2.8 \text{ m/s}$

$\rightarrow V_p = 1.09 \times 2.8 = 3.05 \text{ m/s} \cong 3 \text{ Ok}$.
2. **What’s the maximum discharge** which can be carried by a gutter in a street that has the following characteristics?

- Street longitude slope = 0.5%
- $n = 0.018$
- Cross slope = 4%
- Curb height = 15cm
- Street width = 10m, (3.5 should be clear)

Also find the required curb inlet length to **intercept the entire flow**. And the capacity of a 3 m long curb inlet.

**Solution**

Firstly, we calculate the maximum height the water will rise on the curb:

\[
\text{w} = 3.25\text{m} = y_{\text{max}} \times z \rightarrow y_{\text{max}} = \frac{3.25}{z}
\]

Cross slope = 4% (4: 100) → divided by 4 → the slope will be 1: 25

\[
\rightarrow z = 25 \rightarrow y_{\text{max}} = \frac{3.25}{25} = 0.13\text{m} = 13\text{ cm}
\]

Since 13 cm < 15cm (and 13cm must be maintained)

\[\rightarrow \text{we take } y = 13\text{cm} \text{ as a maximum permitted value.}\]

Now, we calculate the capacity (maximum capacity) of the gutter:

\[
Q_{\text{max}} = 0.38 \times \frac{z}{n} \times S^{0.5} \times y^{8} = 0.38 \times \frac{25}{0.018} \times 0.005^{0.5} \times 0.13^8
\]

\[\rightarrow Q_{\text{max}} = 0.162 \text{ m}^3/\text{s} \checkmark.\]

The equation (given equation) for the curb inlet is:

\[Q = 1.66 \times y^{2/3} \times L\]

But, you should reduce this value by 10% for clogging:

\[\rightarrow Q = 0.9 \times 1.66 \times y^{2/3} \times L = 1.494 \times y^{2/3} \times L\]
To intercept the entire flow in the inlet, the entire maximum values (discharge and height) of the gutter must be in the inlet:

\[ 0.162 = 1.494 \times 0.13^{2/3} \times L \rightarrow L = 0.423 \text{ m} \checkmark. \]

Now, if the curb is 3m long:

\[ Q = 1.494 \times 0.13^{2/3} \times 3 = 1.15 \text{ m}^3/\text{s} \checkmark. \]

**Note:** If grade inlet is used, the equation well differ:

\[ Q = 2.96Ay^{1/2} \text{ but the } \% \text{ of clogging is } 25\% \ (2.96 \times 0.25 \times A \times y^{1/2})\]
1.
In a wastewater treatment plant that served 100,000 inhabitants with average water consumption = 150 L/c/d, you are asked to design a bar screen to protect the equipment of the treatment plant. The bars are 10mm thick, and the openings are 2.5 cm wide and the velocity in the approach channel is 0.65 m/s. **Determine the following:**

a) The cross section of the approach channel assuming the height is double the width.
b) The number of bars.
c) The velocity between bars if the screen 50% clogged with floatable solids.

**Solution**

a) 
To design a bar screen, we design it for **peak flow**, so firstly we calculate the peak flow:

\[ Q_P = Q_{avg(ww)} \times P.f \]

\[ Q_P = \frac{0.8 \times 150 \times 100,000}{1000 \times 24 \times 3600} \times \left(1 + \frac{14}{4 + \sqrt{100}}\right) = 0.278 \text{ m}^3/\text{s} \]

\[ Q_P = Q_{design} = 0.278 = A \times V \text{ (but } v = 0.65 \text{ m/s as given)} \]

\[ \rightarrow A = \frac{0.278}{0.65} = 0.42 \text{ m}^2 \]

\[ A = zw \text{ buy } (z = 2w \text{ as given}) \rightarrow 0.42 = 2w^2 \rightarrow w = 0.46 \text{ m} \checkmark. \]

\[ \rightarrow z = 2 \times 0.46 = 0.92 \text{ m} \checkmark. \]

b) 
To calculate the number of bars, the bars are distributed on the width (w) of the screen, and we are given, the bar thickness is 10mm and each opening (between bars) is 2.5 cm.

Let the number of bars is n, so the number of openings is (n-1), so:

\[ w = n \times t_b + (n - 1) \times w_{opening} \]

\[ \rightarrow 0.46 = n \times 0.01 + (n - 1) \times 0.025 \rightarrow n = 13.86, \text{ use 14 bars} \checkmark. \]
c) To calculate the velocity between bars, we calculate net area of all openings with the 50% clogging (given)

\[
\text{# of bars} = 13.86 \rightarrow \text{# of spacing} = 13.86 - 1 = 12.86
\]

Net Area = 12.86 \times 0.025 \times 0.92 = 0.296 m^2

0.025 is the width of each opening and 0.92 is the screen height.

Net Area (with 50% clogged) = 0.296 \times 0.5 = 0.148 m^2

\[
\rightarrow \text{Velocity between bars } V_b = \frac{Q}{\text{Net Area}} = \frac{0.276}{0.148} = 1.66 \text{ m/sec \checkmark}
\]

2.

In a waste water treatment plant that served 175,000 inhabitant with average waste water production = 120 l/c/d, you are asked to design a grit removal channel to protect the equipment of the treatment plant. One single parabolic grit removal channel is to be designed to remove the grit from this wastewater. Assume \( V_s = 2.5 \text{ cm/sec} \) and \( V_h = 20 \text{ cm/sec} \).

a) Calculate the peak and minimum flow.
b) Find the width of the outlet control section (w). Assume the maximum width of the grit removal channel =1.5 m at Q peak.
c) Find the dimensions of the parabolic cross section and write it in a table.
d) Find the length of the grit removal channel.

Solution

Note:
You are given the wastewater production not water consumption, so you don’t need to multiply it by 0.8.

a)

\[
Q_{avg} = \frac{175,000 \times 120 \times 10^{-3}}{24 \times 60 \times 60} = 0.243 \text{ m}^3/\text{s}.
\]

\[
Q_{peak} = Q_{avg} \times P_f
\]

\[
P_{f_1} = 1 + \frac{14}{4 + \sqrt{P}} = 1 + \frac{14}{4 + \sqrt{175}} = 1.81
\]

\[
P_{f_2} = \frac{5}{P^{0.167}} = \frac{5}{175^{0.167}} = 2.11
\]
P. f\(_3\) = 1.5 + \frac{2.5}{\sqrt{Q_{\text{avg}}(\text{l/sec})}} = 1.5 + \frac{2.5}{\sqrt{243}} = 1.66

So P. f is the maximum value = 2.11

Note: If you are given only one equation for peak factor, use only this equation and check P. f must be greater than 2.

\[ Q_{\text{peak}} = 0.243 \times 2.11 = 0.5127 \text{m}^3/\text{s} \checkmark. \]

\[ Q_{\text{min}} = \frac{Q_{\text{avg}}}{3} = \frac{0.243}{3} = \frac{0.081}{3} \text{ m}^3/\text{s} \checkmark. \]

(when the minimum factor is not given)

b) 
\[ w = \text{????? If } W_{\text{max,peak}} = 1.5 \text{m} \]
\[ Q_{\text{peak}} = 0.5127 \text{m}^3/\text{s} \]
\[ V_h = 20 \text{ cm/ sec} = 0.2 \text{m/sec} \]
\[ A = \frac{2}{3} \times W \times D \text{ (Area of parabolic channel)} \]
\[ A_{\text{peak}} = \frac{Q_{\text{peak}}}{V_h} = \frac{0.5127}{0.2} = 2.5635 \text{m}^2 \quad W=1.5 \text{ m} \]
\[ 2.5635 = \frac{2}{3} \times 1.5 \times D \rightarrow D = 2.5635 \text{ m}. \]
\[ D = \frac{3.1 \times V_c^2}{2g} \rightarrow 2.5635 = \frac{3.1 \times V_c^2}{2 \times 9.81} \rightarrow V_c = 4.028 \text{ m/sec}. \]
\[ d_c = \frac{V_c^2}{g} = \frac{4.028^2}{9.81} = 1.653 \text{ m}. \]
\[ A_{\text{control device}} = d_c \times w = \frac{Q}{V_c} \]
\[ \rightarrow \frac{0.5127}{4.028} = 1.653 \times w \rightarrow w = 0.077 \text{ m} \checkmark. \]

c)
The width of control device (0.077) must be constant to maintain constant \( V_h \).
\[ A_{\text{control device}} = d_c \times w = \frac{Q}{V_c} \rightarrow Q = \frac{0.077 \times V_c^3}{g} \text{ (at any value of Q).} \]

1.
For \( Q_{\text{peak}} = 0.5127 \rightarrow W = 1.5 \text{ m}, D = 2.5635 \text{ m} \text{ (from the upper required).} \]
2. For $Q_{\text{avg}} = 0.243$

\[
Q_{\text{avg}} = 0.243 = \frac{0.077 \times V_c^3}{9.81} \rightarrow V_c = 3.14 \text{ m/s} \rightarrow d_c = \frac{3.14^2}{9.81} = 1 \text{ m}
\]

\[
D = \frac{3.1 \times V_c^2}{2g} = \frac{3.1 \times 3.14^2}{2 \times 9.81} = 1.557 \text{ m} .
\]

\[
A_{\text{avg}} = \frac{Q_{\text{avg}}}{V_h} = \frac{2}{3} \times W \times D \rightarrow \frac{0.243}{0.2} = \frac{2}{3} \times W \times 1.557 \rightarrow W = 1.17 \text{ m} .
\]

3. For $Q_{\text{min}} = 0.081$

\[
Q_{\text{min}} = 0.081 = \frac{0.077 \times V_c^3}{9.81} \rightarrow V_c = 2.177 \text{ m/s} \rightarrow d_c = \frac{2.177^2}{9.81} = 0.483 \text{ m}
\]

\[
D = \frac{3.1 \times V_c^2}{2g} = \frac{3.1 \times 2.177}{2 \times 9.81} = 0.74 \text{ m} .
\]

\[
A_{\text{min}} = \frac{Q_{\text{min}}}{V_h} = \frac{2}{3} \times W \times D \rightarrow \frac{0.081}{0.2} = \frac{2}{3} \times W \times 0.74 \rightarrow W = 0.82 \text{ m} .
\]

<table>
<thead>
<tr>
<th>$Q$</th>
<th>Grit Removal Channel</th>
<th>Outlet Control Device</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W$ (m)</td>
<td>$D$ (m)</td>
</tr>
<tr>
<td>$Q_{\text{min}} = 0.081$</td>
<td>0.82</td>
<td>0.74</td>
</tr>
<tr>
<td>$Q_{\text{avg}} = 0.243$</td>
<td>1.17</td>
<td>1.557</td>
</tr>
<tr>
<td>$Q_{\text{peak}} = 0.5127$</td>
<td>1.5</td>
<td>2.5635</td>
</tr>
</tbody>
</table>

d) \[
\frac{V_h}{V_s} = \frac{L}{D_{\text{peak}}} \rightarrow L = \frac{20}{2.5} \times 2.5635 = 20.504 \text{ m} .
\]
1.
A wastewater flow rate of 19,000 m$^3$/d and $\text{BOD}_{\text{in}}$ 160 mg/l, $\text{COD}_{\text{in}} = 300$ mg/l and $\text{BOD}_{\text{out}}$ of 20 mg/l and to be treated in a completely mix activated-sludge system. The reactor is to operate at a concentration of 3000 mg/l MLSS, and the secondary clarifier to thicken the sludge to 12,000 mg/l. for a mean cell-residence time of 8 days, determine
- the volume of the reactor
- the mass of the solids and the wet volume of sludge wasted each day
- the sludge recycle ratio.
- What is the sludge loading rate (F/M ration)?
- What do you expect to be the COD/BOD ration of the effluent?

Take $K_d = 0.04$ d$^{-1}$, $Y= 0.60$ kg MLSS/kg food

**Solution**

**Given:**

\[
\text{BOD}_{\text{in}} = 160\text{mg/l} \quad \text{BOD}_{\text{out}} = 20\text{mg/l} \\
\theta_c = 8 \text{ days} \quad Y = 0.6 \quad K_d = 0.04 \quad Q = 19,000m^3/\text{day} \\
X = 3000\text{mg/l} \quad X_r = 312,000\text{mg/l} \\
1. \text{Volume} = ??? \\
V = \frac{YQ(S_o - S)\theta_c}{X(1+K_d\theta_c)} = \frac{0.6 \times 19,000 \times 8 \times (160 - 20)}{3000 \times (1 + 0.04 \times 8)} = 3224m^3.
\]

2. a) Mass of solids wasted each day ($Q_wX_r$) = ???

\[
\theta_c = \frac{VX}{Q_wX_r} \rightarrow Q_wX_r = \frac{3224 \times 3000}{8} = 120909.9 \text{ g/day} = 1.2t/day
\]

b) Wet volume of sludge wasted each day ($Q_w$) = ???

\[
Q_w = \frac{Q_wX_r}{X_r} = \frac{120909.9}{12000} = 100.75 \text{ m}^3/\text{day}
\]

3. Sludge recycle ratio ($\frac{Q_r}{Q}$) = ???

\[
Q = 19000 \text{ m}^3/\text{day} \quad Q_r = ???
\]
• Mass balance on aeration tank

\[ Q_r X_r + YQ(S_o - S) - K_d VX = (Q + Q_r)X \]

\[ Q_r \times 12000 + 0.6 \times 19000(160 - 20) - 0.04 \times 3000 \times 3224 = (19000 + Q_r)3000 \]

\[ Q_r = 6199 \text{ m}^3/\text{day} \]

**OR**

• Mass balance on sedimentation tank

\[ (Q + Q_r)X = (Q_w + Q_r)X_r \]

\[ (19000 + Q_r)3000 = (100.75 + Q_r)12000 \]

\[ Q_r = 6199 \text{ m}^3/\text{day} \]

\[ \left(\frac{Q_r}{Q}\right) = \frac{6199}{19000} = 0.326 \]

**Important Note:**

In exam, the formulas sheet contain the following equation to calculate \( Q_r \):

\[ Q_r = Q \frac{X}{X_r - X} \]

But this relation gives \( Q_r \) differ with the one calculated above, because this equation assume \( Q_w = 0 \) and assume also no production of biomass.

So the actual value of \( Q_r \) can be calculated from biomass balance as clarified above, however, if we need approximate value we use the given equation.

But, I calculate \( Q_r \) from biomass balance and you can solve by biomass balance or by the equation.
2. A wastewater flow rate of 21,000 m$^3$/d and BOD influent 280 mg/l and COD in = 350 mg/l to be treated in a completely mix activated-sludge system. The reactor is to operate at a concentration of 3500 mg/l MLSS, and the secondary clarifier to thicken the sludge to 15,000 mg/l for a mean cell-residence time of 7 days. The aeration tank is 5000 m$^3$, determine:

- The efficiency of the reactor
- The mass of the solids as Kg/day of sludge wasted each day
- The sludge recycle as Kg/day.
- What is the sludge loading rate (F/M ratio)?
- After one year of operation, the flow rate became 23,000 m$^3$/day. What do you propose to maintain the same efficiency?

Take $K_d = 0.04$ d$^{-1}$, $Y=0.6$ Kg MLSS/Kg BOD removed

**Solution**

**Givens**

$X=3500$ , $Q=21000$ , $X_r = 15000$

$V=5000$ , $\text{BOD}_{\text{in}} = 280$ $\text{BOD}_{\text{out}}$ = ? ? ?

$K_d = 0.04$ $Y=0.6$ $\theta_c = 7$

4. \[
\left(\frac{\text{COD}}{\text{BOD}}\right)_{\text{out}} = ?? ??
\]

In general: COD is always larger than BOD

@the beginning $\left(\frac{\text{COD}}{\text{BOD}}\right)_{\text{in}} = \frac{300}{160} = 1.875$

الآن عند دخول الماء لحوض التهوية فإن البكتيريا تستهلك المواد العضوية القابلة للتحلل بيولوجيا أي (BOD) بالتالي فإن (COD) تزداد وبالتالي BOD الخارجة تقل وال سورية $\left(\frac{\text{COD}}{\text{BOD}}\right)_{\text{out}} > 2$ (3-4) in general
1. E = ????

\[ BOD_{out} = ??? \]

\[ V = \frac{YQ\theta_c(S_o - S)}{X(1 + \theta_c K_d)} \]

\[ 5000 = \frac{0.6 \times 21000 \times 7 \times (280 - S)}{3500(1 + 0.04 \times 7)} \]

\[ \rightarrow S = BOD_{out} = 26 \text{ mg/l} \]

\[ E = \frac{280 - 26}{280} \times 100 = 90.7\% \]

2. \( Q_w X_r = ??? \)

\[ \theta_c = \frac{V \times X}{Q_w X_r} \]

\[ Q_w X_r = 2500 \text{ Kg/day} \]

3. \( Q_r X_r = ??? \), \( Q_r = ??? \)

\[ Q_r X_r + YQ(S_o - S) - K_d VX = (Q + Q_r)X \]

\[ 15000Q_r + 0.6 \times 21000(280 - 26) - 0.04 \times 5000 \times 3500 \]

\[ = (21000 + Q_r)3500 \]

\[ \rightarrow Q_r = 6173.9 \]

OR

\[ (Q + Q_r)X = (Q_r + Q_w) X_r \]

\[ Q_w = \frac{Q_w X_r}{X_r} = \frac{2500000}{15000} = 166.67 \]

\[ (21000 + Q_r)3500 = (Q_r + 166.67)15000 \]

\[ \rightarrow Q_r = 6173.9 \]
\[ Q_rX_r = 6173.9 \times 15000 \times 10^{-3} = 92608.63 \text{ Kg/day} \]

\[ 4. \frac{F}{M} = ??? \]

\[ \frac{F}{M} = \frac{Q So}{VX} = \frac{21000 \times 280}{5000 \times 3500} = 0.336 \]

4. If \( Q = 21000 \) what do you propose to maintain the same efficiency???

There are two solutions:

a. Increase the value of \( Q_r \) to maintain this efficiency and if we want to calculate the value of \( Q_r \), its calculated as following:

\[ Q_rX_r + YQ(S_o - S) - K_d VX = (Q + Q_r)X \]

\[ 15000Q_r + 0.6 \times 23000(280 - 26) - 0.04 \times 5000 \times 3500 \]

\[ = (23000 + Q_r)3500 \]

\[ \rightarrow Q_r = 6756 \]

الحل الثاني هو زيادة درجة الحرارة مع الابقاء على قيمة \( Q_r \) ثابتة...حيث انه بزيادة درجة الحرارة تنشط البكتيريا وبالتالي تزداد الكفاءة.
3. A trickling filter with 3500 m$^3$ volume and diameter of 32 m has been constructed as a one stage high rate trickling filter to produce a BOD effluent of 55 mg/L. The filter treats 7,500 m$^3$/d with influent BOD = 300 mg/l, water temperature (T) = 20 °C, the recirculation ratio R=1 and Plastic media is used.

Check if the system meets the design criteria of trickling filters and propose any modifications to meet the efficiency if required.

**Solution**

Given:
\[ V = 3500 \text{ m}^3, \quad BOD_{in} = 300 \text{ mg/l}, \quad BOD_{out} = 55 \text{ mg/l} \]
\[ Q = 7500 \text{ m}^3 \text{/day}, \quad R=1, \quad T=20 \]

\[ E_{\text{required}} = \frac{BOD_{in} - BOD_{out}}{BOD_{in}} \times 100 \]

\[ E_{\text{required}} = \frac{300 - 55}{300} \times 100 = 81.67\% \]

This is the required efficiency according to the design criteria.

\[ BOD_{\text{loading rate}} = \frac{BOD_{\text{load}}}{V} = \frac{Q \times BOD_{in}}{V} \]
\[ = \frac{7500 \times 300}{3500} = 642.8 \text{ g/m}^3 \cdot \text{day} = 0.6428 \text{ Kg/m}^3 \cdot \text{day} \]

\[ R=1, \quad F = \frac{1+1}{(1+0.1\times1)^2} = 1.65 \]
\[ E_{design} = \frac{100}{1 + 0.443 \times \left( \frac{BOD_{\text{loading rate}}}{F} \right)^{0.5}} \]

\[ E_{design} = \frac{100}{1 + 0.443 \times \left( \frac{0.6428}{1.65} \right)^{0.5}} = 78.72\% \]

هذه الكفاءة هي التي تم التصميم عليها وهي أقل من الكفاءة المطلوبة 81.67% بالتالي التصميم لا يحقق المتطلبات.

\[ E_{required} < E_{design} \quad \text{Not OK} \]

حتى تصبح كفاءة التصميم مقابلة للكفاءة المطلوبة فإنه يجب زيادة قيمة R حتى تزداد الكفاءة وهنا نجد قيمة R التي تحقق الكفاءة المطلوبة كالتالي:

\[ E_{required} = \frac{100}{1 + 0.443 \times \left( \frac{BOD_{\text{loading rate}}}{R + 1} \right)^{0.5}} \]

\[ 81.67 = \frac{100}{1 + 0.443 \times \left( \frac{0.6428}{R + 1} \right)^{0.5}} \]

\[ R = 3.115 \]

So to meet the required efficiency use \( R \geq 3.115 \) with the same volume.
4. Wastewater with a temperature of 18°C is biologically treated with a trickling filter using plastic media (specific area 150 m²/m³). The existing filter bed is 4.5 m deep and its diameter is 35 m. Future expansion will bring the connected population to 55,000 PE. The specific wastewater production is 120 L/PE.day and the specific BOD is 375 mg/L. The effluent BOD of the treatment plant has to be less than 55 mg BOD/L. **Primary Sedimentation tanks will be used before the trickling filter.**

A. Calculate the future volumetric loading rate in kg BOD/m³.d

B. Is recirculation required? Explain your answer and if necessary calculate recirculation factor R using the NRC equation

C. What will then be the operational hydraulic surface loading rate?

D. It is decided to install a second trickling filter in parallel. What will be the **expected effluent BOD** if R = 2 would be applied.

**Solution**

**Givens:**
- \( T = 18°C \), \( H = 4.5\) m, \( D = 35\) m, population = 55,000 person
- Wastewater production = 120 L/PE.day, \( \text{BOD}_{\text{in}} = 375\) mg/L
- \( \text{BOD}_{\text{out}} = 55\) mg/L, There is (P.S.T) before the trickling filter.

**A. BOD Load rate = ???**

\[
\text{BOD Load rate} = \frac{\text{BOD}_{\text{Load}}}{\text{Volume}}
\]

\[
V = \left( \frac{\pi}{4} \times D^2 \right) \times H = \left( \frac{\pi}{4} \times 35^2 \right) \times 4.5 = 4329.5 \text{ m}^3
\]

\[
\text{BOD}_{\text{Load}} = Q \times \text{BOD}_{\text{in}}
\]

\[
Q = 55,000 \times 120 \times 10^{-3} = 6600 \text{ m}^3/\text{d}.
\]

**Solution**

A. Calculate the future volumetric loading rate in kg BOD/m³.d

B. Is recirculation required? Explain your answer and if necessary calculate recirculation factor R using the NRC equation

C. What will then be the operational hydraulic surface loading rate?

D. It is decided to install a second trickling filter in parallel. What will be the **expected effluent BOD** if R = 2 would be applied.

**Solution**

**Givens:**
- \( T = 18°C \), \( H = 4.5\) m, \( D = 35\) m, population = 55,000 person
- Wastewater production = 120 L/PE.day, \( \text{BOD}_{\text{in}} = 375\) mg/L
- \( \text{BOD}_{\text{out}} = 55\) mg/L, There is (P.S.T) before the trickling filter.

**A. BOD Load rate = ???**

\[
\text{BOD Load rate} = \frac{\text{BOD}_{\text{Load}}}{\text{Volume}}
\]

\[
V = \left( \frac{\pi}{4} \times D^2 \right) \times H = \left( \frac{\pi}{4} \times 35^2 \right) \times 4.5 = 4329.5 \text{ m}^3
\]

\[
\text{BOD}_{\text{Load}} = Q \times \text{BOD}_{\text{in}}
\]

\[
Q = 55,000 \times 120 \times 10^{-3} = 6600 \text{ m}^3/\text{d}.
\]
Sanitary Engineering Discussion

Secondary (Biological) Treatment

\[ \text{BOD}_{\text{Load}} = 6600 \times 243.75 \times 10^{-3} = 1608.75 \text{ Kg/d} \]

\[ \rightarrow \text{BOD}_{\text{Load rate}} = \frac{\text{BOD}_{\text{Load}}}{\text{Volume}} = \frac{1608.75}{4329.5} = 0.37157 \text{ Kg/m}^3 \cdot \text{d} \]

B. Is (R) required?? Why?? If yes → R=??

نلاحظ من المسألة أننا نستطيع حساب قيمة الكفاءة عند درجة حرارة (18) لأن لدينا تركيز (BOD) الداخل والخارج وبالتالي نستطيع إيجاد الكفاءة عند درجة حرارة (20) من الشكل المخصص لذلك والموجود في محاشرات المادة. وبالتالي لمعرفة هل يلزم إعادة إرجاع للمياه أم لا فإن أسهل طريقة هي وضع قيمة (R=0.0) وحساب عندما تكون أكبر من الكفاءة الموجودة أو تساويها فلا يلزم قيمة ل (R) أما لو كانت أقل من الكفاءة الموجودة فيلزم قيمة ل (R) ويتعم حسابها.

\[ E_{18} = \frac{\text{BOD}_{\text{in}} - \text{BOD}_{\text{out}}}{\text{BOD}_{\text{in}}} \times 100 = \frac{243.75 - 55}{243.75} \times 100 = 77.4\% \]

Now, we can calculate the value of (E_{20}) from the chart exist in your lecture notes according the value of (E_{18}) → \( E_{20} \approx 82\% \)

\[ E_{20} = \frac{100}{1 + 0.443 \left( \frac{\text{BOD}_{LR}}{F} \right)^{0.5}}, \quad F = \frac{1 + R}{(1 + 0.1R)^2} \]

\[ E_{20} = 82\%, \; BOD_{LR} = 0.37157 \text{ Kg/m}^3 \cdot \text{d}, \; R = 0.0 \rightarrow F = \frac{1 + 0}{(1 + 0)^2} = 1 \]

At (R = 0.0) → \( E_{20} = \frac{100}{1 + 0.443 \times \left( \frac{0.37157}{1} \right)^{0.5}} = 78.73\% < 82\% \)

نلاحظ أن قيمة الكفاءة المطلوبة لعدم وجود أي إرجاع للماء أقل من القيمة المطلوبة وبالتالي فإن الإرجاع مطلوب وسوف يتم حساب قيمة (R) المقابلة للكفاءة المطلوبة (82%) كما يلي:

\[ 82 = \frac{100}{1 + 0.443 \times \left( \frac{0.37157}{F} \right)^{0.5}} \rightarrow F = 1.513 \]

\[ \rightarrow 1.513 = \frac{1 + R}{(1 + 0.1R)^2} \rightarrow R = 0.7477 \]

C. Surface Loading Rate (SLR) = ???

\[ \text{SLR} = \frac{Q + RQ}{\text{Area}} \text{ range (10} \rightarrow 30) \text{ (see the last example on your lecture notes)} \]

\[ \text{SLR} = \frac{6600 + 0.7477 \times 660}{(\frac{6}{4} \times 35^2)} = 11.99 \text{ (in range) ok} \]
D. If we use two (T.F) in parallel and $R=2 \rightarrow \text{calculate } BOD_{\text{out}}$

The following figure will explain this case:

$$V = 2164.75$$
$$Q = 3300$$
$$Q = 6600$$
$$BOD_{\text{in}} = 243.75$$

$$BOD_{\text{out}} = ???$$

$$V/2$$

$$Q = 3300$$
$$V = 2164.75$$

$$BOD_{\text{Load \_rate}} = 0.37157 \text{ (No change because each V and Q are divided by 2)}$$

$$R = 2 \text{ (Given)} \rightarrow F = \frac{1 + 2}{(1 + 0.1 \times 2)^2} = 2.0833$$

$$E_{20} = \frac{100}{1 + 0.443 \left( \frac{0.37157}{2.0833} \right)^{0.5}} = 84.54\%$$

But the value of $(BOD_{\text{out}})$ is required at $(T = 18^\circ C) \rightarrow E_{18} \approx 80\% \text{ (From chart)}$

$$E_{18} = \frac{BOD_{\text{in}} - BOD_{\text{out}}}{BOD_{\text{in}}} \rightarrow 0.8 = \frac{243.75 - BOD_{\text{out}}}{243.75} \rightarrow BOD_{\text{out}} = 46.75 \text{ mg/}l \quad \circledast$$