Electric Circuits
lecture #1

Eng. Abd Al-Shami H. Abn Jabri

Chapter 1: One e-

The goal of this course is to enable you to determine all variables of any electric circuit.

- Electric Charge Characteristics:
  1. The charge is bipolar (positive, -ve).
  2. Charge exists in discrete quantities. (Multiple of the electron charge)
     electron charge: \( 1.6022 \times 10^{-19} \text{ C} \)
  3. Electrical effects are attributed to both the separation of charge and charge in motion.

- Separation of charge creates \( \Rightarrow \) Voltage.
- Motion of charge creates \( \Rightarrow \) Current.

\[
V = \frac{\Delta W}{\Delta q}, \quad V = \text{voltage (volts)}, \quad \Delta W = \text{energy per unit charge},
\]
\[
\Delta q = \text{energy (joules)},
\]
\[
q = \text{charge (coulombs)}.
\]

\[
i = \frac{\Delta q}{\Delta t}, \quad i = \text{current (amperes)}, \quad \Delta q = \text{rate of charge flow},
\]
\[
\Delta t = \text{time (seconds)}.
\]

- Current will always flow from the higher potential difference to the lower potential difference.
- The assignments of the reference polarity for voltage and the reference direction for current are entirely Arbitrary.
**Passive Sign Convention**

The current flows from the positive terminal to the negative terminal. (outside the battery).

The current flows from negative to positive inside the generator (battery).

*There is NO negative current, if you face any negative value of a current, then the direction you assumed must be inverted.*

**Examples**

\( i = 20 e^{-5000t} \) A, \( t \geq 0 \) so otherwise

1. **Calculate total charge??**

So

\[ I = \frac{dq}{dt} \Rightarrow dq = I \, dt \]

\[ Q = \int_0^\infty I \, dt = \text{area under the curve} \]

\[ Q = \int_{-5000}^{0} 20 \, e^{-5000t} \, dt = 20 \left[ \frac{-e^{-5000t}}{-5000} \right]_{-5000}^{0} = 20 \left[ \frac{-e^{-5000} - e^0} {-5000} \right] \]

\[ Q = \frac{20}{-5000} [1 - 1] = 4,000 \text{ mC} \]

2. **Find \( W \) (energy)??**

**Hint** - \( f(a) \)

**\( f(b) \)**

\[ \int_{a}^{b} f(x) \, dx = \frac{b - a}{2} (f(a) + f(b)) \]

\[ W = \frac{\int_{-100}^{0} (10 - q) \, dq + \int_{200}^{100} (5 + 300 - 200 \cdot 0 + 10) \, dq}{2} = 1650 \text{ J} \]
\[ q = \frac{1}{x^2} - \left( \frac{t}{x} + \frac{1}{x^2} \right) e^{-\alpha t} + C \]

Find max. current? \( \alpha = 0.036795 s^{-1} \)

So \[ I = \frac{dq}{dt} \]

\[ q = \frac{1}{x^2} - \frac{t}{x} e^{\alpha t} - \frac{1}{x^2} e^{\alpha t} \]

\[ \frac{dq}{dt} = 0 - \frac{1}{x^2} \left[ \alpha t + 1 \right] e^{\alpha t} - \frac{1}{x^2} e^{\alpha t} \]

\[ = -\frac{\alpha t}{x^2} e^{\alpha t} + \frac{e^{\alpha t}}{x^2} \]

\[ = \frac{-\alpha t + 1}{x^2} e^{\alpha t} \]

\[ I = t e^{\alpha t} \]

To find max. of \( I \), diff. it and equal with zero

\[ \frac{dI}{dt} = \left( 1 - \alpha t \right) e^{\alpha t} = 0 \]

\[ 1 - \alpha t = 0 \quad \Rightarrow \quad e^{\alpha t} = 0 \]

\[ \alpha t = 1 \quad \Rightarrow \quad t = \frac{1}{\alpha} \]

\[ I = t e^{-\alpha t} = \frac{1}{\alpha} e^{-\alpha/\alpha} = \frac{1}{\alpha} e^{-1} = \frac{1}{\alpha e} \]

\[ I = 9.9 \approx 10 A \]

\[ x \text{ Power and Energy} \]

- If \( P > 0 \), the component absorbs power.
- If \( P < 0 \), the component delivers power.

\[ P = \frac{dw}{dt} \quad P = \text{power (watts)} \]

\[ W = \text{energy (Joules)} \]

\[ t = \text{time (seconds)} \]

\[ P = IU = I^2R = \frac{U^2}{R} \]
Example 2: \( I(t) = 1 \)

\[ V \]

\[ E_1 \]

\[ R_1 \]

\[ 5 \]

\[ 15 \]

\[ 20 \]

\[ t(\text{s}) / \]

\[ 12 \]

\[ 8 \]

\[ \int_0^t I(t) dt \]

(a) Find total charge?

\[ q = \int_0^t I(t) dt \]

As you can notice, the current graph is divided into 3 regions \( R_1, R_2, R_3 \)

\[ \int_{0}^{5} R_1, \quad y_1 = \frac{20 - 14}{t - 5} \]

\[ I_1 = \int_0^5 -1.2t + 2 dt = \left[ -0.6t^2 + 2t + \right]_0^5 = 8.5 \]

\[ \int_{5}^{15} R_2, \quad y_2 = \frac{14 - 8}{5 - 5} = 0.6t + 17 \]

\[ I_2 = \int_5^{15} -0.6t + 17 dt = \left[ -0.3t^2 + 17t \right]_5^{15} = 110 \]

\[ \int_{15}^{20} R_3, \quad y_3 = \frac{8 - 0}{15 - 15} = -1.6t + 32 \]

\[ I_3 = \int_{15}^{20} -1.6t + 32 dt = \left[ -0.8t^2 + 32t \right]_{15}^{20} = 20 \]

\[ q = I \text{ total} = I_1 + I_2 + I_3 = 8.5 + 110 + 20 = 155 \text{ C} \]

(b) Calculate total energy \( W \)?

\[ P = \frac{dW}{dt} \Rightarrow W = \int P \, dt \]

First calculate \( P \) from \( I, V \) graphs and then integrate it to calculate \( W \).
First if $0 < t \leq 5$
\[
p_1 = TV = (-1.2t + 20)(0.2t + 8) = 160 - 5.6t - 0.24t^2
\]
\[
W_1 = \int_5^5 p \, dt = \int_5^5 (160 - 5.6t - 0.24t^2) \, dt = 720 \text{ J}
\]

Second if $5 < t \leq 15$
\[
p_2 = TV = (-0.6t + 17)(0.2t + 8)
\]
\[
W_2 = \int_5^{15} p \, dt = \int_5^{15} (-0.6t + 17)(0.2t + 8) \, dt
\]

Third if $15 < t \leq 20$
\[
p_3 = TV = (-1.6t + 32)(0.2t + 8)
\]
\[
W_3 = \int_{15}^{20} p \, dt = \int_{15}^{20} (-1.6t + 32)(0.2t + 8) \, dt
\]

\[W_{\text{total}} = W_1 + W_2 + W_3\]

\[W \text{ Questions Due to 2/18/2011}\]

1.1, 1.9, 1.15, 1.17, 1.19, 1.21, 1.22, 1.23