The Islamic University of Gaza
Faculty of Engineering
Department of Electrical and Computer Engineering

Final Exam
Electrical Machines (EELE 3351)
Time: 2 Hours

Student’s Name

Student’s Number

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Good Luck
Dr. Assad Abu-Jasser
Tuesday May 27, 2008

All kinds of calculators are allowed
1. The coil on the magnetic core shown has 500 turns. The core has a depth of 5 cm with an air-gap length of 0.07 cm. The remaining dimensions are shown in the figure. Fringing and leakage flux are to be neglected. The core is composed of steel having a relative permeability of $\mu_r=1000$.

a) Calculate the reluctance of the air-gap [3 pts]
b) Calculate the reluctance of the core [5 Pts]
c) Sketch the analogous equivalent circuit of this magnetic arrangement [4 Pts]
d) What current is required to produce a flux density of 0.5 T in the air-gap? [4 pts]
e) What flux density is produced in the right leg if the air-gap is removed? [4 Pts]

\[
\begin{align*}
\mathcal{R}_g &= \frac{l_g}{\mu_0 A_g} = \frac{0.07 \times 10^{-2}}{4 \pi \times 10^{-7} \times 5 \times 5 \times 10^{-4}} = 222817 \text{ A.t/Wb} \\
\mathcal{R}_{r,\text{leg}} &= \frac{l_{r,\text{leg}}}{\mu_r \mu_0 A_{r,\text{leg}}} = \frac{(40-0.07) \times 10^{-2}}{1000 \times 4 \pi \times 10^{-7} \times 5 \times 5 \times 10^{-4}} = 127101 \text{ A.t/Wb} \\
\mathcal{R}_{l,\mu,\text{leg}} &= \frac{l_{l,\mu,\text{leg}}}{\mu_r \mu_0 A} = \frac{115 \times 10^{-2}}{1000 \times 4 \pi \times 10^{-7} \times 10 \times 5 \times 10^{-4}} = 61009 \text{ A.t/Wb} \\
\mathcal{R}_{\text{core}} &= \mathcal{R}_{r,\text{leg}} + \mathcal{R}_{l,\mu,\text{leg}} = 127101 + 188110 = 315211 \text{ A.t/Wb} \\
\mathcal{R}_{\text{total}} &= \mathcal{R}_{\text{core}} + \mathcal{R}_g = 188110 + 222817 = 410927 \text{ A.t/Wb} \\
\phi_g &= B_g \times A_g = 0.5 \times 0.05 \times 0.05 = 0.00125 \text{ Wb} \\
\phi_g &= 1.25 \text{ mWb} \\
\phi &= \frac{Ni}{\mathcal{R}_{\text{total}}} = \frac{500 \times i}{410927} = 1.25 \times 10^{-3} \\
i &= \frac{500}{410927 \times 1.25 \times 10^{-3}} = 1.027 \text{ A} \\
\mathcal{R}_{r,\text{leg}} &= \frac{l_{r,\text{leg}}}{\mu_r \mu_0 A_{r,\text{leg}}} = \frac{40 \times 10^{-2}}{1000 \times 4 \pi \times 10^{-7} \times 5 \times 5 \times 10^{-4}} = 127324 \text{ A.t/Wb} \\
\mathcal{R}_{\text{core}} &= \mathcal{R}_{r,\text{leg}} + \mathcal{R}_{l,\mu,\text{leg}} = 127324 + 188110 = 315434 \text{ A.t/Wb} \\
\mathcal{R}_{\text{total}} &= \mathcal{R}_{\text{core}} = 188324 \text{ A.t/Wb} \\
\phi_{\text{core}} &= \frac{Ni}{\mathcal{R}_{\text{total}}} = \frac{500 \times 1.027}{188324} = 0.00273 \text{ Wb} \\
\phi_{\text{core}} &= 2.73 \text{ mWb} \\
\phi &= \frac{\phi_{\text{core}}}{A_{r,\text{leg}}} = \frac{0.00273}{25 \times 10^{-4}} = 1.09 \text{ T}
\end{align*}
\]
A 100-kVA, 1000/100-V, 60-Hz single-phase transformer gave the following test result:

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<th>Test</th>
<th>Voltage</th>
<th>Current</th>
<th>Power</th>
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<tr>
<td>Open-circuit test (HV side open)</td>
<td>100 V</td>
<td>6 A</td>
<td>400 W</td>
</tr>
<tr>
<td>Short-circuit test (LV side shorted)</td>
<td>50 V</td>
<td>100 A</td>
<td>1800 W</td>
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a) Find the parameters of the approximate equivalent circuit referred to the HV side [10 Pts]

Open circuit test

\[
P_{oc} = V_{oc} I_{oc} \cos \theta_{oc} \quad \cos \theta_{oc} = \frac{400}{100 \times 6} = 0.667 \quad \theta_{oc} = 48.19^\circ
\]

\[
\sin \theta_{oc} = 0.745 \quad Q_{oc} = V_{oc} I_{oc} \sin \theta_{oc} = 100 \times 6 \times 0.745 = 104.5 \text{ VA}
\]

\[
100^2 / P_{oc} = 1000 \quad R_{cl} = 25 \Omega \quad X_{ML} = V_{oc}^2 / 100 = 95.72 \Omega
\]

\[
a = \frac{100}{100} = 10 \quad R_{cH} = 10^2 \times 25 = 2500 \Omega \quad X_{MHL} = 10^2 \times 95.72 = 9527 \Omega
\]

Short circuit test

\[
P_{sc} = V_{sc} I_{sc} \cos \theta_{sc} \quad \cos \theta_{sc} = \frac{1800}{50 \times 100} = 0.36 \quad \theta_{sc} = 68.9^\circ
\]

\[
\sin \theta_{sc} = 0.933 \quad Q_{sc} = V_{sc} I_{sc} \sin \theta_{sc} = 50 \times 100 \times 0.933 = 4665 \text{ VA}
\]

\[
R_{eqH} = \frac{P_{sc}}{I_{sc}^2} = \frac{1800}{100^2} = 0.18 \Omega \quad X_{eqH} = \frac{Q_{sc}}{I_{sc}^2} = \frac{4665}{100^2} = 0.4665 \Omega
\]

\[
V_2' = 100 \times 10 = 1000 \text{ V} \quad I_{2'-\text{rated}} = \frac{100000}{100} = 1000 \text{ A}
\]

\[
V_1 = V_1' + I_1' (R_{eqH} + jX_{eqH}) = 1000 \times 100 \times 36.87^\circ (0.18 + j 0.4665)
\]

\[
VR\% = \frac{V_2' - V_1'}{V_2'} \times 100 = \frac{987.58 - 1000}{1000} \times 100 = -1.242\%
\]

\[
P_{out} = 100 \times 0.8 = 80 \text{ kW}
\]

\[
P_{core} = \frac{V_1^2}{R_{cH}} = \frac{987.58^2}{2500} = 390.13 \text{ W} \approx P_m = 400 \text{ W}
\]

\[
P_{in} = P_{out} + P_{cu} + P_{core} = 80000 + 1800 + 390.13 = 82190.13 \text{ W}
\]

\[
\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{82190.13}{82190.13} \times 100 = 97.34\%
\]
3. A three-phase, 400-hp, 480-V, 60-Hz, 8-pole, Δ-connected synchronous motor has a synchronous reactance of 1.0 Ω and negligible armature resistance. Ignore friction, windage, and core losses of this motor

a) What are the magnitudes and phase angles of \( E_A \) and \( I_A \) if the motor is supplying 400 hp at 0.8 lagging power factor? [8 pts]

b) Calculate the torque produced by the motor and the torque angle \( \delta \) [6 pts]

c) If \( E_A \) is increased by 15 percent while the power output remains constant, what is the new magnitude of the armature current? What is the new power factor? [6 pts]

\[
\begin{align*}
\text{(a)} & \\
\text{P}_{\text{in}} & = P_{\text{out}} = 400 \times 746 = 298.4 \text{ kW} \\
I_L & = \frac{P}{\sqrt{3}V_T \cos \theta} = \frac{298.4 \times 1000}{\sqrt{3} \times 480 \times 0.8} = 488.65 - 36.87^\circ A \\
I_A & = \frac{I_L}{\sqrt{3}} = \frac{488.65}{\sqrt{3}} = 285.37 - 36.87^\circ A \\
E_A & = V_\phi - jX_s I_A = 480 \left[ 0^\circ - j 1.0 \times 285.37 \right] = 385 - 32.55^\circ V \\
n_S & = \frac{120f}{P} = \frac{120 \times 60}{8 \pi \times 90} = 900 \text{ rpm} \\
\omega & = \frac{2\pi \times n_S}{8} = \frac{2\pi \times 900}{60} = 94.25 \text{ rad/s} \\
\tau & = \frac{P}{2\pi \times n_S} = \frac{298.4 \times 1000}{94.25} = 3166 \text{ N.m} \\
E_A & = 1.15 \times 385 = 442.75 V \\
P & = \frac{3V_\phi E_A}{X_s} \sin \delta_2 = \sin^{-1} \left( \frac{298.4 \times 1000 \times 1.0}{3 \times 480 \times 442.75} \right) = 27.9^\circ \\
I_{A2} & = \frac{V_\phi - E_A}{jX_s} = \frac{480 \left[ 0^\circ - 442.75 \right]}{1.0} = 207.18 - j 88.71 A \\
I_{A2} & = 207.18 - j 88.71 = 225.37 - 23.18^\circ A \\
pf & = \cos 23.18 = 0.92 \text{ lagging}
\end{align*}
\]
4. A 208-V, four-pole, 60-Hz, Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent-circuit components are

\[
\begin{align*}
R_1 &= 0.220 \, \Omega \\
R_2 &= 0.127 \, \Omega \\
X_M &= 15.0 \, \Omega \\
X_1 &= 0.430 \, \Omega \\
X_2 &= 0.430 \, \Omega \\
P_{\text{mech}} &= 300 \, \text{W} \\
P_{\text{misc}} &= 0 \\
P_{\text{core}} &= 200 \, \text{W}
\end{align*}
\]

For a slip of 0.05 find

a) The line current \( I_L \) drawn by the motor [5 Pts]

b) The power converted from electrical to mechanical form \( P_{\text{conv}} \) [5 Pts]

c) The maximum developed torque the machine can produce [6 Pts]

d) The overall machine efficiency at a slip of 0.05 [4 Pts]