Answer all the following 4 questions.
Read the questions carefully and write your answers under the corresponding questions.
Be neat and organized.
Show all the necessary work. Correct answer without sufficient explanation might not get full credit.

Q.1  (a) Let $A$ and $B$ be sets. Prove that if $A \subseteq B$, then $A - B = \phi$.

(b) Let $A$, $B$, and $C \neq \phi$ be sets. Prove that $(A \times C) \subseteq (B \times C)$ if and only if $A \subseteq B$. 

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<thead>
<tr>
<th>Name:</th>
<th>Q.1</th>
<th>Q.2</th>
<th>Q.3</th>
<th>Q.4</th>
<th>Total</th>
</tr>
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<td>Instructor : Dr. Ayman Hashem Sakka</td>
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<td>10</td>
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Instructor : Dr. Ayman Hashem Sakka

Total: 40
Q.2 (a) Let $A$, $B$, $C$, and $D$ be sets. Prove that $(A \cap B) - (C \cup D) = (A - C) \cap (B - D)$

(b) Let $X = \{1, 2, 3, 4, 5, \pi, e, \sqrt{2}\}$ and let $P = \{\{1, 3, 5\}, \{2, \sqrt{2}\}, A\}$ be a partition of $X$. Find $A$.

(c) Let $P$ be partition of a nonempty set $X$. Prove that if $P = \{X\}$, then $X/P = X \times X$. 

\[ \]
Q.3 Let $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : 2x + y = 3k \text{ for some } k \in \mathbb{Z}\}$.

(a) Prove that $R$ is an equivalence relation.

(b) Find the set of all equivalence classes $\mathbb{Z}/R$. 
Q.4 Choose the correct answer for each of the following items:

(i) For any sets $A$ and $B$, one of the following statements is false
   
   (a) $(A \cup B) \subseteq A$.
   (b) $(A \cap B) \subseteq A$.
   (c) $(A \cap B) \subseteq B$.
   (d) $A \subseteq (A \cup B)$.

(ii) For any sets $A$ and $B$, one of following statements is true
   
   (a) $A \nsubseteq B \Rightarrow (A \cap B) = \emptyset$.
   (b) $(A \times B) = (B \times A)$.
   (c) $(A \cap B)' = A' \cap B'$.
   (d) $A \cup \emptyset = A$.

(iii) For any sets $A$ and $B$, one of the following statements is false
   
   (a) $A \subseteq B \Rightarrow B' \subseteq A'$.
   (b) $A' \subseteq B' \Rightarrow B \subseteq A$.
   (c) $A \subseteq B \Rightarrow A' \subseteq B'$.
   (d) $A \cup A' = B \cup B'$.

(iv) Let $R$ be a relation from $A$ to $B$. Then one of the following statements is true
   
   (a) $\text{Dom}(R^{-1}) = B$.
   (b) $\text{Dom}(R^{-1}) = A$.
   (c) $\text{Dom}(R^{-1}) \subseteq A$.
   (d) $\text{Dom}(R^{-1}) \subseteq B$.

(v) Let $E$ be an equivalence relation on a set $X \neq \emptyset$. Then one of the following statements is false
   
   (a) $(\forall x \in X)(x/E \neq \emptyset)$.
   (b) $(x, y) \in E \Rightarrow x/E \cap y/E = \emptyset$.
   (c) $x/E \cap y/E \neq \emptyset \Rightarrow (x, y) \in E$.
   (d) $(x, y) \in E \Rightarrow x/E = y/E$. 

4