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GOOD LUCK
1. a) What is the underlying idea of the fuzzy set theory? (1 PT)

b) Probability theory has existed since, at least, 1930. Why do we need fuzzy set theory in addition? Is it not all covered by probability theory? What are the main differences between these two theories? Can we just replace probability theory with the theory of fuzzy sets? Or do the two theories complement each other? Give explanation/examples for your opinion. (2 PT)

c) A fuzzy set \( S \) on \( Z \) is shown to the right. (2 PT)
   i) What is the Support of \( S \)?
   ii) Height of \( S \)?
   iii) Is \( S \) convex?
   iv) Is \( S \) normal?
   v) What is the cardinality of \( S \)?
   vi) What is the level set of \( S \)?
2. Methane biofilters can be used to oxidize methane using biological activities. It has become necessary to compare performance of two test columns, A and B. The methane outflow level at the surface, in nondimensional units of \( X = \{50, 100, 150, 200\} \), was detected and is tabulated below against the respective methane inflow into each test column. The following fuzzy sets represent the test columns:

\[
A = \left\{ \frac{0.15}{50} + \frac{0.25}{100} + \frac{0.5}{150} + \frac{0.7}{200} \right\} \\
B = \left\{ \frac{0.2}{50} + \frac{0.3}{100} + \frac{0.6}{150} + \frac{0.65}{200} \right\}
\]

Calculate the union, intersection, and the difference for the test columns. (3 PT)
3. We want to compare two sensors based upon their detection levels and gain settings. For a universe of discourse of gain settings, \( X = \{0, 20, 40, 60, 80, 100\} \), the sensor detection levels for the monitoring of a standard item provides typical membership functions to represent the detection levels for each of the sensors; these are given below in standard discrete form:

\[
\begin{align*}
S_1 &= \left\{ \frac{0}{0} + \frac{0.5}{20} + \frac{0.65}{40} + \frac{0.85}{60} + \frac{1.0}{80} + \frac{1.0}{100} \right\} \\
S_2 &= \left\{ \frac{0}{0} + \frac{0.45}{20} + \frac{0.6}{40} + \frac{0.8}{60} + \frac{0.95}{80} + \frac{1.0}{100} \right\}
\end{align*}
\]

Find the following membership functions using standard fuzzy operations: 

(a) \( \mu_{S_1 \cup S_2}(x) \)
(b) \( \mu_{S_1 \cap S_2}(x) \)
(c) \( \mu_{\neg S_1}(x) \)
(d) \( \mu_{\neg S_2}(x) \)
(e) \( \mu_{S_1 \cup \neg S_1}(x) \)
(f) \( \mu_{S_1 \cap \neg S_1}(x) \)
4. In the field of computer networking there is an imprecise relationship between the level of use of a network communication bandwidth and the latency experienced in peer-to-peer communications. Let $X$ be a fuzzy set of use levels (in terms of the percentage of full bandwidth used) and $Y$ be a fuzzy set of latencies (in milliseconds) with the following membership functions:

$$X = \left\{ \begin{array}{c} 0.2 + \frac{0.5}{20} + \frac{0.8}{40} + \frac{1.0}{60} + \frac{0.6}{80} + \frac{0.1}{100} \end{array} \right\}$$

$$Y = \left\{ \begin{array}{c} 0.3 + \frac{0.6}{0.5} + \frac{0.9}{1.5} + \frac{1.0}{4} + \frac{0.6}{8} + \frac{0.3}{20} \end{array} \right\}$$

(a) Find the Cartesian product represented by the relation $R = X \times Y$. Now, suppose we have a second fuzzy set of bandwidth usage given by $Z$. Find $S = Z_{1 \times 6} \circ R_{6 \times 6}$.

(b) Using max–min composition; 

(c) Using max–product composition.
5. List the different parts of a fuzzy controller. Describe the roles and purposes of the different parts. (Mamdani & Sugeno Control) (3 PT)
6. In mechanics, the energy of a moving body is called kinetic energy. If an object of mass \( m \) (kilograms) is moving with a velocity \( v \) (meters per second), suppose we model the mass and velocity as inputs to a system (moving body) and the energy as output, then observe the system for a while and deduce the following two rules of inference based on our observations:

**Rule 1:** IF \( x_1 \) is small and \( x_2 \) is high velocity, THEN \( y \) is medium energy

**Rule 2:** IF \( x_1 \) is large mass or \( x_2 \) is medium velocity, THEN \( y \) is high energy.

Let input \( (i) = 0.35 \) kg (mass) and input \( (j) = 55 \) m/s (velocity), find the output using a Mamdani implication (use Centroid, and Weighted average method for defuzzification).