CHAPTER 1

MEASUREMENTS AND VECTORS
1.1 UNITS AND STANDARDS

Any physical quantity must have, besides its numerical value, a standard unit. It will be meaningless to say that the distance between Gaza and Jerusalem is 80 because 80 kilometers is different from 80 meters or 80 miles, where kilometer, meter, and mile are standards for length known all over the world. Several systems of units are used in physics: The most common system among them is the Systém International (French for International System) abbreviated SI. The SI units for the seven-fundamental physical quantities are *kilogram* for mass, *meter* for length, *second* for time, *kelvin* for temperature, *ampere* for electric current, *candela* for luminous intensity, and *mole* for the amount of substance. All other physical quantities are derived from these basic quantities. In mechanics all quantities are derived from the three-fundamental quantities: *mass*, *length*, and *time*.

**One meter** is defined as the distance traveled by light in vacuum during a time of 1/299,792,458 second.

**One kilogram** is defined as the mass of a specific platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures in France.

**One second** is defined as the time required for a cesium-133 atom to undergo 9,192,631,770 vibrations.

The other two systems are the Gaussian system and the British engineering system. The three systems and their standard units for mass, length, and time are listed in Table 1.1. Because of their units the first two systems are sometimes called the *mks* system (the first letters of meter, kilogram, and second) and the *cgs* system (the first letters of centimeter, gram, and second) respectively.

If different unit systems are used in a physical equation one system should be chosen and the quantities with the other unit systems must be converted to the chosen system according to the conversion factors of Table 1.2.
Table 1.1 Units of Length, Mass, and Time in Different systems.

<table>
<thead>
<tr>
<th>Systems</th>
<th>Length</th>
<th>Mass</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>meter (m)</td>
<td>kilogram (kg)</td>
<td>second (s)</td>
</tr>
<tr>
<td>cgs</td>
<td>Centimeter (cm)</td>
<td>gram (g)</td>
<td>second (s)</td>
</tr>
<tr>
<td>British</td>
<td>foot (ft)</td>
<td>slug</td>
<td>second (s)</td>
</tr>
</tbody>
</table>

Table 1.2 Conversion Factors.

<table>
<thead>
<tr>
<th>Length</th>
<th>Mass</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m = 10^2 cm = 3.28 ft</td>
<td>1 kg = 10^3 g</td>
<td>1 year = 3.16 x 10^7 s</td>
</tr>
<tr>
<td>1 mi = 5280 ft = 1.61 km</td>
<td>1 slug = 14.6 kg</td>
<td>1 day = 8.64 x 10^4 s</td>
</tr>
<tr>
<td>1 yd = 3 ft = 36 in</td>
<td>1 u = 1.66 x 10^-27 kg</td>
<td></td>
</tr>
</tbody>
</table>

1.2 DIMENSIONAL ANALYSIS

Dimension in physics gives the physical nature of the quantity, whether it is a length (L), mass (M), or time (T). All other quantities in mechanics can be expressed in terms of these fundamental quantities. For example, the dimension of velocity is length divided by time and denoted by \( [v] = \frac{L}{T} \). The dimensions of some physical quantities are listed in Table 1.3.

Table 1.3 Dimensions and units of some physical quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Dimension</th>
<th>Unit (SI, cgs, British)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>L^2</td>
<td>m^2, cm^2, ft^2</td>
</tr>
<tr>
<td>Volume</td>
<td>L^3</td>
<td>m^3, cm^3, ft^3</td>
</tr>
<tr>
<td>Velocity</td>
<td>L/T</td>
<td>m/s, cm/s, ft/s</td>
</tr>
<tr>
<td>Acceleration</td>
<td>L/T^2</td>
<td>m/s^2, cm/s^2, ft/s^2</td>
</tr>
<tr>
<td>Force</td>
<td>ML/T^2</td>
<td>Newton (N), dyne, pound</td>
</tr>
<tr>
<td>Energy</td>
<td>ML^2/T^2</td>
<td>Joul (J), erg, ft.lb</td>
</tr>
<tr>
<td>Power</td>
<td>ML^2/T^3</td>
<td>Watt (W), erg/s,</td>
</tr>
</tbody>
</table>
**Example 1.1** Show that the equation \( x = v_0 t + \frac{1}{2} at^2 \) is dimensionally correct.

**Solution** Since \([x] = L\), \([v_0] = \frac{L}{T}\), and \([a] = \frac{L}{T^2}\),

Then

\[
L = \frac{L}{T} T + \frac{L}{T^2} T^2 = L
\]

### 1.3 VECTORS AND SCALARS

A **scalar** is the physical quantity that has magnitude only, for example, time, volume, mass, density, energy, distance, temperature. On the other hand a **vector** is the physical quantity that has both magnitude and direction, for example, displacement, velocity, acceleration, force, area.

![Figure 1.1](image)

**Figure 1.1** Equality of vectors and the negative of a vector. Note that \( A > B \).

The vector quantity will be distinguished from the scalar quantity by typing it in boldface, like \( \mathbf{A} \). In write handing the vector quantity is written with an arrow over the symbol, such as, \( \vec{A} \). The magnitude of the vector \( \mathbf{A} \) will be denoted by \( |\mathbf{A}| \), or simply the
italic type \( A \). Geometrically, The vector quantity is represented by a straight line and an arrow at one end of the line. The end at which the arrow is attached is called the **head of the vector**, while the other end is called the **tail of the vector**. The length of the line is proportional to the magnitude of the vector and the arrow points toward its direction (see Figure 1.1).

**Equality of Two Vectors** Any two vectors are said to be equal if they have the same magnitude and point in the same direction, regardless of their location.

![Figure 1.1](image1.png)

Addition (graphical method). To add vector \( \mathbf{A} \) to another vector \( \mathbf{B} \), (1) in a graph paper, draw vector \( \mathbf{A} \) with its magnitude represented by a proper scale. (2) Draw vector \( \mathbf{B} \) according to the same scale such that its tail starts at the head of vector \( \mathbf{A} \). (3) Draw the resultant vector \( \mathbf{R}=(\mathbf{A}+\mathbf{B}) \) from the tail of vector \( \mathbf{A} \) to the head of vector \( \mathbf{B} \), (see Figure 1.2). This method is known as the triangle method.

Another method is the parallelogram method. In this method the tail of the two vectors \( \mathbf{A} \) and \( \mathbf{B} \) are coincident and the resultant vector is the diagonal of the parallelogram formed by the two vectors \( \mathbf{A} \) and \( \mathbf{B} \). As it is clear from Figure 1.2, the addition of vectors obey the commutation and association laws, i.e., respectively.
\[ A + B = B + A \] \hspace{1cm} (1.1)

and

\[ A + (B + C) = (A + B) + C \] \hspace{1cm} (1.2)

**Negative of a Vector** The vector \(-A\) is a vector with the same magnitude as the vector \(A\) but points in opposite direction.

**Subtraction** Subtracting vector \(B\) from vector \(A\) is the same as adding \(-B\) to \(A\), i.e.,

\[ A - B = A + (-B) \] \hspace{1cm} (1.3)

The arithmetic processes of vectors using the graphical method described above are not easy. In the following sections we describe easier techniques that involve algebra.

**1.4 UNIT VECTOR**

\[ \text{Figure 1.3} \] The unit vectors \(i\), \(j\), and \(k\) are pointed toward the positive \(x\), \(y\), and \(z\) axes, respectively.

From its name, a unit vector is a vector, with specific direction, having a magnitude of unity without any units or dimensions. In Cartesian coordinates three unit vectors are adopted to specify the
positive directions of the three-axes. These are: \( \mathbf{i} \) for the positive \( x \)-axis, \( \mathbf{j} \) for the positive \( y \)-axis and \( \mathbf{k} \) for the positive \( z \)-axis, as shown in Figure 1.3. This means that if a vector \( \mathbf{A} \) is directed along the positive \( x \)-axis with a magnitude of \( A \), this vector can be written as \( \mathbf{A} = A \mathbf{i} \). Similarly, a vector \( \mathbf{B} \), directed along the positive \( y \)-axis and has a magnitude \( B \) is written as \( \mathbf{B} = B \mathbf{j} \). The vector \( \mathbf{C} = C \mathbf{k} \) means that it has a magnitude of \( C \) and directed along the positive \( z \)-axis. The minus sign in front of any vector indicates the opposite direction of that vector, i.e. \( -\mathbf{i} \) refers to the negative \( x \)-axis, and so for the other two unit vectors.

### 1.5 COMPONENTS OF VECTORS

Any vector \( \mathbf{A} \) in a plane can be represented by the sum of two vectors, one parallel to the \( x \)-axis (\( \mathbf{A}_x \)), and the other parallel to the \( y \)-axis (\( \mathbf{A}_y \)) as shown in Figure 1.4, i.e.

\[
\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y.
\]  

(1.4)

Since, \( \mathbf{A}_x = A_x \mathbf{i} \) and \( \mathbf{A}_y = A_y \mathbf{j} \) we can write Equation 1.4 as

\[
\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j}.
\]
\[ \mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} \]  

(1.5)

where \( A_x \) and \( A_y \) are the \( x \)-component and the \( y \)-component of the vector \( \mathbf{A} \), respectively. From Figure 1.4 it is clear that

\[ A_x = A \cos \theta \quad \text{and} \quad A_y = A \sin \theta \]  

(1.6)

The magnitude of the vector \( \mathbf{A} \) is given by

\[ A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2} \]  

(1.7)

In general any vector \( \mathbf{A} \) can be resolved into three components as

\[ \mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \]  

(1.8)

where \( A_z \) is called the \( z \)-component of the vector \( \mathbf{A} \). Note that if \( \mathbf{A} = 0 \), then \( A_x = 0 \), \( A_y = 0 \), and \( A_z = 0 \).

**Example 1.2** A vector \( \mathbf{A} \) lying in the \( x-y \) plane has a magnitude \( A = 50.0 \) units and is directed at an angle of \( 120^\circ \) to the positive \( x \)-axis, as shown in Figure 1.5. What are the rectangular components of this vector?

**Solution** From Equation (1.6) we have

\[ A_x = A \cos \theta = 50 \cos 120^\circ = -25.0 \text{ units} \]

and

\[ A_y = A \sin \theta = 50 \sin 120^\circ = 43.3 \text{ units} \]
1.6 ADDING VECTORS

To add vectors analytically proceed as follow:
1) Resolve each vector into its components according to suitable coordinate axes.
2) Add, algebraically, the $x$-components of the individual vectors to obtain the $x$-component of the resultant vector. Do the same thing for the other components, i.e., if

$$
A = A_x i + A_y j + A_z k
$$

and

$$
B = B_x i + B_y j + B_z k ,
$$

then the resultant vector $R$ is

$$
R = A + B = R_x i + R_y j + R_z k
$$

or

$$
R = (A_x + B_x) i + (A_y + B_y) j + (A_z + B_z) k \tag{1.9}
$$

Example 1.3 If $A = 4i + 3j$ and $B = -3i + 7j$, find the resultant vector $R = A + B$.

Solution From the given information we have

$$
A_x = 4, A_y = 3, B_x = -3, \text{ and } B_y = 7
$$

Now from Equation (1.9) we have

$$
R_x = A_x + B_x = 1.00 ,
$$

and
\[ R_y = A_y + B_y = 10.0 \]

so we can write

\[ \mathbf{R} = i + 10j \]

**Example 1.4** A particle undergoes three consecutive displacements given by \( \mathbf{d}_1 = (i + 3j - k) \) cm, \( \mathbf{d}_2 = (2i - j - 3k) \) cm, and \( \mathbf{d}_3 = (-i + j) \) cm. Find the resultant displacement of the particle.

**Solution** \( \mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 \), or

\[ \mathbf{R} = (1 + 2 - 1)i + (3 - 1 + 1)j + (-1 - 3 + 0)k \]
\[ = (2i + 3j - 4k) \text{ cm} \]

**Example 1.5** A particle undergoes the following consecutive displacements: 4.3 m southeast, 2.4 m east, and 5.2 m north. Find the magnitude and the direction of the resultant vector.

**Solution** If we denote the three displacements by \( \mathbf{d}_1, \mathbf{d}_2, \text{ and } \mathbf{d}_3 \), respectively, we get the vector diagram shown in Figure 1.6. According to the coordinates system chosen, the three vectors can be written as

\[ \mathbf{d}_1 = (4.3 \cos 45)i - (4.3 \sin 45)j \]
\[ = (3.04i - 3.04j) \text{ m} \]

\[ \mathbf{d}_2 = 2.4i \text{ m}, \text{ and } \mathbf{d}_3 = 5.2j \text{ m} \]
now,
\[ \mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 \]
\[ = (3.04 + 2.40 + 0)i + (-3.04 + 0 + 5.20)j \]
\[ = (5.44i + 2.16j) \text{ m} \]

The magnitude of \( \mathbf{R} \) is
\[ R = \sqrt{R_x^2 + R_y^2} = \sqrt{(5.44)^2 + (2.16)^2} = 5.85 \text{ m} \]

To find the direction of a vector it is enough to determine the angle the vector makes with a specific axis. The angle \( \theta \) makes with the \( x \)-axis is
\[ \theta = \tan^{-1} \left( \frac{R_y}{R_x} \right) = \tan^{-1} \left( \frac{2.16}{5.44} \right) = 21.66^\circ \]

**Example 1.6** If \( \mathbf{B} = 4i - j \), find the vector \( \mathbf{A} \) such that \( \mathbf{A} + \mathbf{B} = 5i \).

**Solution** Since \( \mathbf{A} + \mathbf{B} = 5i \) then,
\[ A_x + B_x = 5, \text{ and } A_y + B_y = 0 \]
so we have
\[ A_x = 5 - B_x = 5 - 4 = 1 \]
and
\[ A_y = -B_y = 1 \]

This leads to
\[ \mathbf{A} = i + j \]
1.7 PRODUCTS OF VECTORS

1- Multiplying a Vector by a Scalar: The product of a vector $\mathbf{A}$ and a scalar $\lambda$ is a new vector with a direction similar to that of $\mathbf{A}$ if $\lambda$ is positive but opposite to the direction of $\mathbf{A}$ if $\lambda$ is negative. The magnitude of the new vector $\lambda \mathbf{A}$ is equal to the magnitude of $\mathbf{A}$ multiplied by the absolute value of $\lambda$, i.e.,

$$\lambda \mathbf{A} = \lambda A_x \mathbf{i} + \lambda A_y \mathbf{j} + \lambda A_z \mathbf{k}$$ (1.10)

2- Scalar (Dot) Product: The scalar product of two vectors $\mathbf{A}$ and $\mathbf{B}$, denoted by $\mathbf{A} \cdot \mathbf{B}$, is a scalar quantity defined by,

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$ (1.11)

with $\theta$ is the smallest angle between the two vectors. Since $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors perpendicular to each other, and using Equation (1.11), we can write

$$\mathbf{i} \cdot \mathbf{j} = \mathbf{i} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{j} = 0$$ (1.12a)

$$\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$$ (1.12b)

Equations 1.12 imply that the scalar product of the vectors $\mathbf{A}$ and $\mathbf{B}$ can be written as

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$ (1.13)

Example 1.7 If $\mathbf{A} = \mathbf{i} - 2 \mathbf{j} + 3 \mathbf{k}$ and $\mathbf{B} = 2 \mathbf{i} + 3 \mathbf{j} - 2 \mathbf{k}$, find the angle between the two vectors $\mathbf{A}$ and $\mathbf{B}$

Solution From Equation (1.11) we have
\[
\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB}.
\]

But from Equation (1.7) we have
\[
A = \sqrt{1+4+9} = 3.74, \text{ and }
\]
\[
B = \sqrt{4+9+4} = 4.12.
\]

Using Equation (1.13) we find
\[
\mathbf{A} \cdot \mathbf{B} = 2 - 6 - 6 = -10.0.
\]

So we now have
\[
\cos \theta = \frac{-10}{(3.74)(4.12)} = -0.69,
\]
or
\[
\theta = \cos^{-1}(-0.69) = 134^\circ
\]

**Example 1.8** Consider the two vectors \( \mathbf{A} \) and \( \mathbf{B} \) given in the previous example (example 1.7), find \( 2\mathbf{A} \cdot \mathbf{B} \)

**Solution**

\( 2\mathbf{A} = 2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} \),

now from Equation 1.13, we obtain

\[
(2\mathbf{A}) \cdot \mathbf{B} = 4 - 12 - 12 = -20.0
\]

Note that if one multiply the product \( \mathbf{A} \cdot \mathbf{B} \) by 2, or multiply \( \mathbf{A} \cdot (2\mathbf{B}) \) the same result will be achieved, that is

\[
2(\mathbf{A} \cdot \mathbf{B}) = (2\mathbf{A}) \cdot \mathbf{B} = \mathbf{B} \cdot (2\mathbf{A})
\]
3- Vector (cross) Product: The vector product of two vectors \( \mathbf{A} \) and \( \mathbf{B} \), written as \( \mathbf{A} \times \mathbf{B} \), is a third vector \( \mathbf{C} \) with a magnitude given by

\[
C = |\mathbf{A} \times \mathbf{B}| = AB \sin \theta
\]

(1.14)

The vector \( \mathbf{C} \) is perpendicular to the plane of \( \mathbf{A} \) and \( \mathbf{B} \). Since there are two directions perpendicular to a given plane, the right hand rule is used to decide to which direction the vector \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \) is directed. The rule states that the four fingers of the right hand are pointed along \( \mathbf{A} \) and then curled toward \( \mathbf{B} \) through the smaller angle between \( \mathbf{A} \) and \( \mathbf{B} \). The thumb then gives the direction of \( \mathbf{C} \). From this rule one can verify that

\[
\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}
\]

(1.15)

and

\[
i \times i = j \times j = k \times k = 0 \quad (1.16a) \\
i \times j = k \quad (1.16b) \\
j \times k = i \quad (1.16c) \\
k \times i = j \quad (1.16d)
\]

From Equations (1.16) we can write

\[
\mathbf{A} \times \mathbf{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\
= (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}
\]

or, equivalently
Example 1.9 Find the vector product of the two vectors given in the previous example (example 1.7)

Solution

\[
A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}
\]

\begin{align*}
A \times B &= \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\
&= \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & 3 & -2 \end{vmatrix} \\
&= (4-9)i + (6+2)j + (3+4)k = -5i + 8j + 7k
\end{align*}

PROBLEMS

1.1 Show that the equation \( v^2 = v_0^2 + 2ax \) is dimensionally correct, where \( v \) and \( v_0 \) represent velocities, \( x \) is a distance, and \( a \) is an acceleration.

1.2 The period of a simple pendulum with the unit of time is given by \( T = 2\pi \sqrt{\frac{l}{g}} \) where \( l \) is the length of pendulum, and \( g \) is the acceleration due to gravity. Show that the equation is dimensionally correct.

1.3 A vector \( \mathbf{A} \) in the \( x-y \) plane is 12 units long and directed as shown in Figure 1.7, Determine the \( x \) and the \( y \) components of the vector.

1.4 Vector \( \mathbf{A} \) is 5 units in length and points along the positive \( x \)-axis. Vector \( \mathbf{B} \) is 7 units in length and points along the negative \( y \)-axis.
   a) Find \( \mathbf{A} + \mathbf{B} \).
   b) Find the magnitude and the direction of \( \mathbf{A} + \mathbf{B} \).

1.5 Vector \( \mathbf{A} \) has a magnitude of 4 units and makes an angle of 37° with the positive \( x \)-axis. Vector \( \mathbf{B} \) has a magnitude of 6 units and makes an angle of 120° with the positive \( x \)-axis.
a) Find the $x$-component and the $y$-component of each vector.
b) Find $\mathbf{A} - \mathbf{B}$

1.6 A car is driven east for a distance of 30 km. It next driven north-east for a distance of 20 km, and then in a direction of $30^\circ$ north of west for a distance of 40 km. Draw the vector diagram of the three displacements and find their resultant (magnitude and direction).

1.7 Consider the three vectors shown in Figure 1.10. If $A=8$ m, $B=5$ m, and $C=12$ m, find
a) the $x$ and the $y$-components of each vector,
b) the magnitude and the direction of the resultant vector.

![Figure 1.10 (Problem 1.7)](image)

1.8 Find the magnitude and the direction of the resultant of the three displacement having components (-3,3) m, (2,4) m, and (5,-1) m.

1.9 Two vectors are given by $\mathbf{A}=3\mathbf{i}+3\mathbf{j}$, and $\mathbf{B}=-2\mathbf{i}+7\mathbf{j}$. Find
a) $\mathbf{A} + \mathbf{B}$,
b) $\mathbf{A} - \mathbf{B}$,
c) $|\mathbf{A}|$,
d) $|\mathbf{B}|$.

1.10 You are given the two vectors $\mathbf{A}=2\mathbf{i}+3\mathbf{j}$, and $\mathbf{B}=6\mathbf{i}+8\mathbf{j}$. Find the magnitudes and the directions of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{A} + \mathbf{B}$, and $\mathbf{A} - \mathbf{B}$.

1.11 If $\mathbf{A}=2\mathbf{i}+3\mathbf{j}-\mathbf{k}$, and $\mathbf{B}=\mathbf{i}+3\mathbf{j}$, and $\mathbf{C}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, find
a) \( \mathbf{A} + \mathbf{B} + \mathbf{C} \)

b) \( \mathbf{A} - \mathbf{B} - \mathbf{C} \)

1.12 Given the two vectors \( \mathbf{A} = 4\mathbf{i} - 3\mathbf{j} \), and \( \mathbf{B} = 2\mathbf{i} + 4\mathbf{j} \). Find the vector \( \mathbf{C} \) such that \( \mathbf{A} + \mathbf{B} + \mathbf{C} = 0 \).

1.13 Find a unit vector parallel to \( \mathbf{A} = 6\mathbf{i} - 8\mathbf{j} \).

1.14 Given the two vectors \( \mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 7\mathbf{k} \), and \( \mathbf{B} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} \), find

a) \( 2\mathbf{A} + \mathbf{B} \)

b) \( \mathbf{A} - 2\mathbf{B} \).

1.15 Find the scalar product \( \mathbf{A} \cdot \mathbf{B} \) of the two vectors \( \mathbf{A} \), and \( \mathbf{B} \) in problem 1.11.

1.16 Find the angle between the two vectors \( \mathbf{A} = -2\mathbf{i} + 3\mathbf{j} \), and \( \mathbf{B} = 3\mathbf{i} - 4\mathbf{j} \).

1.17 For what values of \( \lambda \) are the two vectors \( \mathbf{A} = \mathbf{i} + 2\mathbf{j} + \lambda\mathbf{k} \), and \( \mathbf{B} = 2\lambda\mathbf{i} - 2\mathbf{j} + 2\lambda\mathbf{k} \) perpendicular to each other.

1.18 If \( \mathbf{A} = 2\mathbf{i} + 6\mathbf{j}, \mathbf{A} - \mathbf{B} = \mathbf{C}, \) and \( \mathbf{A} + \mathbf{B} = 2\mathbf{C} \) find \( \mathbf{B} \) and \( \mathbf{C} \).

1.19 Using the scalar product, prove the law of cosine (the law is stated in appendix D).

1.20 Given \( \mathbf{A} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} \), and \( \mathbf{B} = \mathbf{i} + 3\mathbf{j} \). Find

a) \( \mathbf{A} \times \mathbf{B} \)

b) \( \mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) \)

1.21 Three vectors are given by \( \mathbf{A} = \mathbf{i} + \mathbf{j} + \mathbf{k} \), \( \mathbf{B} = \mathbf{i} + 2\mathbf{j} \), and \( \mathbf{C} = 2\mathbf{i} - \mathbf{k} \). Find

a) \( \mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) \)

b) \( \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) \)
c) \( \mathbf{A} \times (\mathbf{B} + \mathbf{C}) \)

1.22 The two vectors \( \mathbf{A} \), and \( \mathbf{B} \) represent concurrent sides of a parallelogram as shown in Figure 1.11. Show that the area of the parallelogram is \( |\mathbf{A} \times \mathbf{B}| \).

1.23 Find a unit vector perpendicular to the plane of the two vectors given in problem 1.17.

1.24 Two vectors of magnitudes \( A \) and \( B \) make an angle \( \theta \) with each other when placed tail to tail (see Figure 1.12). Prove that the magnitude of their sum, \( R \) is given by

\[
R = \sqrt{A^2 + B^2 + 2AB \cos \theta}
\]

1.25 Prove that for any three vectors \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \)

\[
\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}
\]