Water Treatment
EENV 4331

Lecture 5: Filtration

Dr. Fahid Rabah
5. Filtration in water Treatment

5.1 Definition of Sedimentation:
Filtration is a solid–liquid separation process in which the liquid passes through a porous medium to remove as much fine suspended solids as possible.

5.2 Locations of filtration tanks in water treatment:
Filtration tanks are used in all types of water treatment plants except for disinfection treatment plants. See Figures 5.1 through 5.4 illustrating the location of filtration tanks.
River Water

Pre-Sedimentation

Screen

Coagulation

Flocculation

Sedimentation

Filtration

Disinfection

Storage

Distribution

Figure 5.1: Filtration Treatment Plant (River Water)
Figure 5.2: Filtration Treatment Plant
Figure 5.3: Softening Treatment Plant Single stage softening
Figure 5.4: Aeration Treatment Plant (iron and manganese removal plant)
5. Filtration in water Treatment

5.3 Need for filtration:

- Settling is not sufficient to remove all particles and flocs from water.
- Filtration Needed for fine particles not removed by sedimentation.
- Filters can also capture Giardia cysts, viruses, and asbestos fibers.
- Typical overflow qualities from sedimentation tanks range from 1 to 10 NTU.
- Filtration, usually rapid sand filtration, is then employed for further “polishing”, i.e. to get the turbidity to lower than 0.5 NTU (as required by legislation).
- Rapid sand filtration after prior sedimentation is the most common configuration worldwide.
5. Filtration in water Treatment

5.4 Types of filters used in water treatment:
Granular material filters are the most used types of filters in water treatment. Usually sand, anthracite, and Garnet.
There are three types of granular filters:
1. Single –medium filters :
   one type of media is used: either sand or anthracite

2. Dual-media filters: two types of media is used usually sand and anthracite

3. Multimedia filters: three types of media are used usually sand, anthracite, and Garnet
Most famous filters in water treatment are Rapid Sand Filters.
5. Filtration in Water Treatment

5.5 Geometry and Components of Rapid Sand Filter:

- Rapid sand filters are always rectangular tanks.
- Figures 5.5 to 5.10 show typical Rapid sand filters used in water treatment.
- Main components of Rapid sand filter are:
  1. A concrete tank
  2. Filter media
  3. Under drain system
  4. Backwash system: pressurized water and air lines
Figure 5.5: Rapid sand filter components
Figure 5.6a:
Rapid sand filter components: with gravel and perforated pipes under drain system
Figure 5.6b:
Rapid sand filter components: with gravel and perforated pipes under drain system
Figure 5.7:
Rapid sand filter components: with gravel and perforated pipes under drain system
Figure 5.8:
Rapid sand filter components: with ducts under-drain system
Figure 5.9:
Rapid sand filter components: with nozzle under-drain system
5. Filtration in water Treatment

Figure 5.10: Rapid sand filter perforated slab and nozzle under-drain system
5. Filtration in water Treatment

Figure 5.11: Nozzle used in Rapid sand filter under-drain system
5. Filtration in water Treatment

5.6 Operation of Rapid Sand Filter:

There are two modes of operation of Rapid sand filter

- Filtration mode (see Figure 5.12)
- Backwashing mode (see Figure 5.13)
5. Filtration in water Treatment

5.7 Filtration mode:

- Water flows downward through a bed of sand and gravel
- Particles are captured on and between sand grains
- Filtered water is collected in the under drain, sent to disinfection
5. Filtration in water Treatment

5.8 Backwash mode:

• Sand is backwashed when
  – It becomes clogged, or
  – Turbidity of filtered water gets too high

• During backwash, water is pumped upwards through the sand bed
5. Filtration in water Treatment

• Sand becomes “fluidized”, and particles are flushed from the sand
• Dirty backwash water is pumped into a settling pond, and either
  – Recycled back into plant, or
  – Disposed
• Backwashing can consume 1% to 5% of a plant’s production
Figure 5.12: Rapid sand filter during filtration
Figure 5.13: Rapid sand filter during backwashing
5. Filtration in water Treatment

5.9 Filter media properties

Figure 5.14: filter media grain distribution

Cross section through ideal filter uniformly graded from coarse to fine from top to bottom
5. Filtration in water Treatment

-These filters use sand and crushed anthracite coal on a graded gravel base.

-Media layers are arranged in a course to fine gradation in the direction of flow, which allows greater depth of penetration of floc particles.

-Multimedia filters are selected with specific gravities so that moderate intermixing between media layers occurs during backwashing.
5. Filtration in water Treatment

5.10 Filter media properties

The filter media is characterized by two main parameters: the effective size and the uniformity coefficient.

**Effective size of the filter media**

The effective size of the media is the diameter that 10% of the filter media is less than it size and is denoted as $d_{10}$.

**Uniformity coefficient of the filter media**

$$U = \frac{d_{60}}{d_{10}}$$

$U$ = Uniformity coefficient

$d_{60}$ = sieve size that passes 60% by weight

$d_{10}$ = sieve size that passes 10% by weight

- $d_{60}$ and $d_{10}$ are found by sieve analysis of the media to be used in the filter.

- Another important sieve size is $d_{90}$ that is used to calculate the backwash rate.
Figure 5.15: Rapid sand media layers
Figure 5.16: Rapid sand media layers
### 5. Filtration in water Treatment

**Table 5.1.** Single-Medium Filter Characteristics for Water Treatment

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>Range</th>
<th>Typical</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sand medium:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>24–30</td>
<td>27</td>
</tr>
<tr>
<td>(mm)</td>
<td>(610–760)</td>
<td>(685)</td>
</tr>
<tr>
<td>Effective size, mm</td>
<td>0.35–0.70</td>
<td>0.60</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
<td>&lt;1.7</td>
<td>&lt;1.7</td>
</tr>
<tr>
<td><strong>Anthracite medium:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>24–30</td>
<td>27</td>
</tr>
<tr>
<td>(mm)</td>
<td>(610–760)</td>
<td>(685)</td>
</tr>
<tr>
<td>Effective size, mm</td>
<td>0.70–0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
<td>&lt;1.75</td>
<td>&lt;1.75</td>
</tr>
<tr>
<td><strong>Filtration rate:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gpm/ft²</td>
<td>2–5</td>
<td>4</td>
</tr>
<tr>
<td>(ℓ/s-m²)</td>
<td>(1.36 3.40)</td>
<td>(2.72)</td>
</tr>
</tbody>
</table>
5. Filtration in water Treatment

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range</td>
</tr>
<tr>
<td>Anthracite:</td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>18–24</td>
</tr>
<tr>
<td>(mm)</td>
<td>(460–610)</td>
</tr>
<tr>
<td>Effective size, mm</td>
<td>0.9–1.1</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
<td>1.6–1.8</td>
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<tr>
<td>Sand:</td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>6–8</td>
</tr>
<tr>
<td>(mm)</td>
<td>(150–205)</td>
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<tr>
<td>Effective size, mm</td>
<td>0.45–0.55</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
<td>1.5–1.7</td>
</tr>
<tr>
<td>Filtration rate:</td>
<td></td>
</tr>
<tr>
<td>gpm/ft$^2$</td>
<td>3–8</td>
</tr>
<tr>
<td>($\ell$/s-m$^2$)</td>
<td>(2.04–5.44)</td>
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</table>
5. Filtration in water Treatment

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<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>Range</th>
<th>Typical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anthracite:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>16.5–21</td>
<td>18</td>
</tr>
<tr>
<td>(mm)</td>
<td>(420–530)</td>
<td>(460)</td>
</tr>
<tr>
<td>Effective size, mm</td>
<td>0.95–1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
<td>1.55–1.75</td>
<td>&lt;1.75</td>
</tr>
<tr>
<td>Sand:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>6–9</td>
<td>9</td>
</tr>
<tr>
<td>(mm)</td>
<td>(150–230)</td>
<td>(230)</td>
</tr>
<tr>
<td>Effective size, mm</td>
<td>0.45–0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
<td>1.5–1.65</td>
<td>1.60</td>
</tr>
<tr>
<td>Garnet:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>3–4.5</td>
<td>3</td>
</tr>
<tr>
<td>(mm)</td>
<td>(75–115)</td>
<td>(75)</td>
</tr>
<tr>
<td>Effective size, mm</td>
<td>0.20–0.35</td>
<td>0.20</td>
</tr>
<tr>
<td>Uniformity coefficient</td>
<td>1.6–2.0</td>
<td>&lt;1.6</td>
</tr>
<tr>
<td>Filtration rate:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>gpm/ft²</td>
<td>4–10</td>
<td>6</td>
</tr>
<tr>
<td>(ℓ/s-m²)</td>
<td>(2.72–6.80)</td>
<td>(4.08)</td>
</tr>
</tbody>
</table>
5. Filtration in water Treatment

Figure 5.17:
Head loss and effluent turbidity increase with time during filtration
5. Filtration in water Treatment

Figure 5.14: Head loss in rapid sand filter during filtration cycle
Figure 5.14: Head loss in rapid sand filter
Head loss in a clean filter

Carmen–Kozeny equation:

\[
\frac{h}{L} = \frac{k \mu (1 - \varepsilon)^2}{g \rho \varepsilon^3} \left( \frac{V}{A} \right)^2 \frac{v}{\phi d}
\]

Where,

\( k \) = dimensionless coefficient, 5 for sand, 6 for anthracite;
\( v \) = filtration rate \( \text{m}^3/\text{m}^2 \cdot \text{d} \), or filtration velocity \( \text{m/d} \).
\( A \) = the grain surface area;
\( A_s \) = surface area of the sand filter;
\( V \) = the grain volume;
\( \varepsilon \) = filter porosity; around 0.40 for sand filter
\( \Phi \) = shape factor; 1 for spherical particles, 0.70 for sand;
\( \mu \) = dynamic viscosity; \( \text{N.s/m}^2 \)
\( \rho \) = water density; \( \text{kg/m}^3 \)
\( h \) = head loss in clean filter, m
Example 5.1:

A dual media filter is composed of 0.30 m anthracite (mean particle size 0.20mm) that is placed over a 0.60 m layer of sand (mean particle size 0.70mm) with a filtration rate of 9.78 m/h. Assume the grain sphericity $\phi = 0.75$ and porosity $(\varepsilon) = 0.40$ for both. Estimate the headloss in the clean filter at $15^\circ$C.

A. Head loss in the anthracite layer:

$$h = 0.30 \times 6 \times \frac{0.00113 \times (1 - 0.40)^2 \left(\frac{6}{0.75 \times 0.002}\right)^2 \times 0.00272}{9.81 \times 1000 \times 0.40^3} = 0.0508m$$

B. Head loss in the sand layer:

$$h = 0.60 \times 5 \times \frac{0.00113 \times (1 - 0.40)^2 \left(\frac{6}{0.75 \times 0.007}\right)^2 \times 0.00272}{9.81 \times 1000 \times 0.40^3} = 0.6918m$$

B. Head loss in the sand layer:

$$h_{total} = 0.0508 + 0.6918 = 0.743 \ m$$
Head loss during filtration
(None clean filter)

\[(h_l)_t = v (a + b V_{filtered})\]

Where,

- \(v\) = filtration rate \(m^3/m^2.d\), or filtration velocity \(m/d\).
- \(a, b\) = coefficients depending on the filter media properties;
- \(V_{filtered}\) = filtered volume per unit area of filter since last backwash; \(m^3/m^2\)
- \((h_l)_t\) = head loss at any time \(t\), m
Example 5.2:

A filter has a head loss of 0.30 m when clean (newly washed), and 1.30 m after 24 hrs of filtration at a rate of 1.5 L/s.m². Estimate the head loss both immediately after backwash and 10 hrs later, if the filtration rate is changed to 2 L/s.m².

A. Estimate the values of a and b:

\[
0.30 = \frac{1.5}{1000} (a + b \times 0)
\]

\[
1.30 = \frac{1.5}{1000} \left( a + b \times \frac{1.5}{1000} \times 24 \times 3600 \right)
\]

By solving the 2 equations simultaneously, \( a = 200, \ b = 5.14 \)

B. Calculate head loss for the new flow rate:

\[
H_0 = \frac{2}{1000} (200 + b \times 0) = 0.40 \text{ m}
\]

\[
H_{10} = \frac{2}{1000} \left( 200 + 5.14 \times \frac{2}{1000} \times 10 \times 3600 \right) = 1.88 \text{ m}
\]
Filtration hydraulics:

1) Apply Bernoulli equation between point 1 and 2

\[
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{v_{pipe}^2}{2g} + Z_2 + (h_i)\_
\]

\[
\frac{p_1}{\gamma} = \text{Zero}, \quad \frac{v_1^2}{2g} = \text{Zero}, \quad Z_1 = H, \quad \frac{p_2}{\gamma} = h, \quad Z_2 = \text{Zero}
\]
Filtration hydraulics Calculations

\[(h_i)_t = \text{head loss at any time (t)}\]

Substituent these values in the Bernoulli equation:

\[H = h_t + \frac{v_{\text{pipe}}^2}{2g} + (h_i)_t \ldots (1)\]

\[h_t = \text{pressure head available just at the outlet of the filter at any time(t)}\]

\[(h_i)_t = (h_i)_{\text{Anthracite}} + (h_i)_{\text{sand}} + (h_i)_{\text{garnet}} + (h_i)_{\text{under drain}}\]

\[(h_i)_t \begin{bmatrix} \text{sand} \\ \text{garnet} \\ \text{anthracite} \end{bmatrix} \rightarrow \text{use carmen Kozeny equation for clean filter}\]

\[(h_i)_t \begin{bmatrix} \text{sand} \\ \text{garnet} \\ \text{anthracite} \end{bmatrix} \rightarrow \text{use the equation for dirty filter}\]

\[(h_i)_t = v(a + bV_{\text{filtered}})\]
Filtration Hydraulics Calculations

\[(h_i)_{\text{under drain}} \rightarrow \text{if the under drain is gravel, use Carmen Kozeny equation for clean filter}\]

\[\rightarrow \text{if the under drain is slab and nozzel use the value given by the Manufacturer.}\]

\[L = L_{\text{anthracite}} + L_{\text{sand}} + L_{\text{garnet}} = \text{Total filter media depth}\]

\[H = d + L + L_u - \frac{D_{\text{pipe}}}{2} \quad \ldots (2)\]

\[d = \text{water depth above filter media during filtration.}\]

From 1 and 2:

\[h_t = d + L + L_u - \frac{D_{\text{pipe}}}{2} - \frac{v^2_{\text{pipe}}}{2g} - (h_i)_{\text{t}} \quad \ldots (3)\]

\[d = \frac{v^2_{\text{pipe}}}{2g} + 1.5 \ (h_i)_{\text{clean}} \quad \ldots (4)\]

From 3 and 4:

\[h_t = L + L_u - \frac{D_{\text{pipe}}}{2} + 1.5 \ (h_i)_{\text{clean}} - (h_i)_{\text{t}} \quad \ldots (5)\]
Calculations of filter backwash rate

The backwash flow rate is calculated using the following equations:

\[ v_b = \frac{\mu}{\rho d_{90}} \left(1135.69 + 0.0408 G_n\right)^{0.5} - \frac{33.7 \mu}{\rho d_{90}} \]

\[ G_n = \frac{d_{90}^3 \rho (\rho_s - \rho) g}{\mu^2} \]

\[ (v_b)_{design} = 1.3 v_b \]

Where,
- \( v_b \) = backwash rate \( m^3/m^2.d \)
- \( d_{90} \) = sieve size that passes 90% by weight
- \( \mu \) = dynamic viscosity; N.s/m²
- \( \rho \) = water density; kg/m³
- \( \rho_s \) = filter particles density; kg/m³
- \( G_n \) = Galileo number, dimensionless
- \( g \) = gravitational acceleration, m/s²
Calculations of filter expansion

The expansion during backwash is calculated using the following equations:

\[ L_e = L \left( \frac{1 - \varepsilon}{1 - \varepsilon_e} \right) \]

\[ \varepsilon_e = \left( \frac{(v_b)_{design}}{v_s} \right)^{0.22} \]

Where,
\( L \) = bed depth during filtration, m
\( L_e \) = expanded bed depth, m
\( \varepsilon_e \) = expanded bed porosity, dimensionless
\( \varepsilon \) = bed porosity during filtration, dimensionless
\( v_s \) = settling velocity of the filter particles, m/s
Headloss during backwashing is calculated using the following equation:

\[
h = \frac{L_e \left(1 - \varepsilon_e \right)(\rho_s - \rho)}{\rho}
\]

**Where,**
- \(L_e\) = expanded bed depth, m
- \(\varepsilon_e\) = expanded bed porosity, dimensionless
- \(\rho\) = water density; kg/m\(^3\)
- \(\rho_s\) = filter particles density; kg/m\(^3\)
Backwash hydraulics:

1) Apply Bernoulli equation between point 1 and 2

\[
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{v_{pipe}^2}{2g} + Z_2 + h_l
\]

but \( \frac{p_1}{\gamma} = \text{zero}, Z_1 = H, \frac{v_1^2}{2g} = \text{zero}, Z_2 = \text{zero}, \frac{p_2}{\gamma} = h \)
Backwash hydraulics Calculations

So → \[ H = h + \frac{v_{pipe}^2}{2g} + h_l \] ...(1)

\[ h_l = \frac{10.70 \text{L}}{D_{pipe}^{1.852}} \left( \frac{Q_b}{C_{HW}} \right), \] (Hazen Williams Equation)

\[ Q_b = (V_b)_{design} \times A_{filter} \] ...(a)

\[ A_{pipe} = \frac{Q_b}{v_{pipe}} \] ..................................(b)

\[ A_{pipe} = \frac{\pi D_{pipe}^2}{4} \] ..................................(c)

\[ A_{pipe} = A_{filter} \times \frac{(v_b)_{design}}{v_{pipe}} \] ...................................(d)

Note: →"H" is the height of the Backwash tank.

→ "h" is the available head just at the inlet of the filter.

2) Apply Bernoulli equation between point 2 and 3

L= (pipe length from a→b + b→c)

\[ C_{HW} = 140 - 150 \]

\[ D_{pipe} = \text{backwash pipe diameter} \]

\[ Q_b = \text{backwash flow} \]

\[ v_{pipe} \text{ typical range is } 1 - 1.5 \text{ mls} \]
Backwash hydraulics Calculations

\[
\frac{p_2}{\gamma} + \frac{v_{\text{pipe}}^2}{2g} + Z_2 + \frac{p_3}{\gamma} + \frac{v_{\text{b}}^2}{2y} + Z_3 + (h_i)_{\text{filter}} + (h_i)_{\text{under drain}}
\]

\[
\left[\frac{p_2}{\gamma} = h, Z_2 = \text{Zero}, \frac{p_3}{\gamma} = Z_3 = l_{\text{filter}} - \frac{D_{\text{pipe}}}{2}\right]
\]

\[
\Rightarrow H + \frac{v_{\text{pipe}}^2}{2g} = \frac{v_{\text{b}}^2}{2g} + Z_3 + \left[l_{\text{filter}} - \left(\frac{D_{\text{pipe}}}{2}\right)\right] + (h_i)_{\text{filter}} + (h_i)_{\text{under drain}}
\]

\[
H = \frac{v_{\text{h}}^2}{2g} - \frac{v_{\text{pipe}}^2}{2g} + \left[l_{\text{filter}} - \left(\frac{D_{\text{pipe}}}{2}\right)\right] + (h_i)_{\text{filter}} + (h_i)_{\text{under drain}}
\]

\[
(h_i)_{\text{filter}} = \Sigma \text{ of head loss loss in each filter layer}
\]

\[
= (h_i)_{\text{garnet}} + (h_i)_{\text{sand}} + (h_i)_{\text{Anthracite}}
\]

\[
(h_i)_{\text{sand}} \Rightarrow \text{use the equation } \Rightarrow h_i = \frac{L_e (1 - \varepsilon_e) (\rho_s - \rho)}{\rho} \text{ garnet anthracite}
\]

\[
(h_i)_{\text{under drain}} \Rightarrow \text{if the under drain is slab and nozzel type, } (h_i)_{\text{under drain}} \text{ is given by the manufacturer.}
\]

\[
\Rightarrow \text{if the under drain is grarel use carmen Kozeuy equation to calculateit.}
\]
Backwash hydraulics Calculations

\[ h_i = L_u \times \frac{k\mu(1 - \varepsilon)^2}{g \rho \varepsilon^3} \times \left( \frac{V}{A} \right)^2 \cdot \nu \]

\[ L_{filter} = L_u + L_{e} + L_{w} = 0.5m + L_{e} + 0.5m \]

\[ = L_{e} + 1.0m \]

\[ L_{e} = (L_{e})_{sand} + (L_{e})_{garnet} + (L_{e})_{Anthracite} \]
Filtration hydraulics:

1) Apply Bernoulli equation between point 1 and 2

\[ \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{v_{pipe}^2}{2g} + Z_2 + (h_t) \]

\[ \frac{p_1}{\gamma} = \text{Zero,} \quad \frac{v_1^2}{2g} = \text{Zero,} \quad Z_1 = H, \quad \frac{p_2}{\gamma} = h, \quad Z_2 = \text{Zero} \]
\[(h_i)_t = \text{head loss at any time (t)}\]

Substituent these values in the Bernoulli equation:

\[H = h_t + \frac{v^2_{\text{pipe}}}{2g} + (h_i)_t \ldots (1)\]

\[h_t = \text{pressure head available just at the outlet of the filter at any time (t)}\]

\[(h_i)_t = (h_i)_{\text{Anthracite}} + (h_i)_{\text{sand}} + (h_i)_{\text{garnet}} + (h_i)_{\text{under drain}}\]

\[\begin{bmatrix} \text{sand} \\ \text{garnet} \\ \text{anthracite} \end{bmatrix} \rightarrow \text{use carmen Kozeny equation for clean filter}\]

\[\begin{bmatrix} \text{sand} \\ \text{garnet} \\ \text{anthracite} \end{bmatrix} \rightarrow \text{use the equation for dirty filter}\]

\[(h_i)_t = v(a + bV_{\text{filtered}})\]
Filtration hydraulics Calculations

\[(h_i)_{\text{under drain}} \rightarrow \text{if the under drain is gravel, use Carmen Kozeny equation for clean filter} \]
\[\rightarrow \text{if the under drain is slab and nozzel use the value given by the Manufacturer.} \]

\[L = L_{\text{anthracite}} + L_{\text{sand}} + L_{\text{garnet}} = \text{Total filter media depth} \]

\[H = d + L + L_u - \frac{D_{\text{pipe}}}{2} \quad \cdots (2)\]

\[d = \text{water depth above filter media during filtration.} \]

From 1 and 2:

\[h_t = d + L + L_u - \frac{D_{\text{pipe}}}{2} - \frac{v_{\text{pipe}}^2}{2g} - (h_i)_t \quad \cdots (3)\]

\[d = \frac{v_{\text{pipe}}^2}{2g} + 1.5 (h_i)_{\text{clean}} \quad \cdots (4)\]

From 3 and 4:

\[h_t = L + L_u - \frac{D_{\text{pipe}}}{2} + 1.5 (h_i)_{\text{clean}} - (h_i)_t \quad \cdots (5)\]