Do all the following four questions. Show all the necessary work to receive full credit for each answer. Be neat and organized.

1. Consider the complex number $1 + i$.
   
   (a). Find $\arg(1 + i)$.

   (b). Find the polar representation of $1 + i$.

   (c). Use De Moivre’s Theorem to express $(1 + i)^{11}$ in the form $x + iy$, where $x, y \in \mathbb{R}$. 
(d). Find the principal value of \((1 + i)^i\) and write it in the form \(x + iy\).

2. (a). Prove that \(\overline{e^z} = e^{\overline{z}}\) \(\forall z \in \mathbb{C}\).

(b). Use part (a) and the definition of \(\cos z\) to show that
\[
\cos \overline{z} = \cos \overline{z} \quad \forall z \in \mathbb{C}
\]
(c). Find the image $f(A)$ of the set

$$A = \{ z = x + iy : x \geq 0, -\pi \leq y < \pi \}$$

under the mapping $f(z) = e^z$ and sketch it in the complex plane.

(d). Consider the set $f(A)$ that you found in part (c). Mark each of the following statements by (T) if it is true and by (F) if it is false:

(i). $f(A)$ is closed.
(ii). $f(A)$ is bounded.
(iii). $f(A)$ is connected.
(iv). 0 an accumulation point of $f(A)$.
(v). $i$ is a boundary point of $f(A)$. 

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3. (a). Use $\epsilon - \delta$ definition of the limit to show that

$$\lim_{z \to 0} \frac{z \text{Re} z}{z} = 0.$$ 

(b). Using Cauchy-Riemann equations and the definition of a derivative, determine the points $z$ where the function $f(z) = z \text{Re} z$ has a derivative.
4. (a). Find a pws parametrization of the pws curve $\gamma$, given in the Figure below, joining 0 to 1 and 1 to $1+i$.

(b). Calculate the integral $\int z \, dz$ where $\gamma$ is the pws curve given in part (a).

(c). Calculate the integral $\int_{\gamma^*} z \, dz$ where $\gamma^*$ is the line segment joining $1+i$ to 0 in above Figure.