Exercise 3.1.2

a) Since n is a positive number, the while loop in this algorithm will run forever, therefore this algorithm is not finite.

b) Since algorithm is not effective since the line “m := 1/n” cannot be executed when n=0, which will eventually be the case. It can also be argued that this algorithm is not finite: if a line in the algorithm cannot be completed, the algorithm as a whole cannot be completed. It can also be argued that the algorithm lacks correctness since the “m := 1/n” line will also keep the algorithm from arriving at a correct answer.

c) The value of j is never set in the algorithm, so the algorithm lacks definiteness. Without knowing the initial value of j, the behavior of this algorithm is undetermined.

d) The only line in the algorithm is ambiguous, how does the algorithm decide which value (a or b) to assign to x? Without knowing how this decision is made, the behavior of this algorithm is undetermined; therefore, this algorithm lacks definiteness.
Section 3.1

real x, \text{ \texttt{exp}}(x, n)

\begin{align*}
\text{Let } n &= 0 \\
\text{let } \text{output} &= \frac{1}{x} \\
\text{for } i = 1 \text{ to } |n| \\
\quad &\text{output} = \text{output} \times x \\
\quad &\text{if } (n < 0) \\
\quad &\text{output} = \frac{1}{\text{output}} \\
\text{output} &= x^n
\end{align*}

Pseudocode

procedure \text{exp} (x: real, n: integer)

output := 1

for (i := 1 \text{ to } |n|)

output := output \times x

if (n < 0)

output := 1/output

output := x^n

\text{Description: -}

* Set the output to 1 to be an initial value and the output if \( n = 0 \).
* Find \( x^n \) by multiply \( x \) by itself \( |n| \) times.
* Check if \( n \) is negative and if true make the output is \( 1/\text{output} \) of the previous step.
Procedure \text{minmax} (a_1, a_2, \ldots, a_n)

\text{max} := a_1 \\
\text{min} := a_1 \\
\text{for (i = 1 to n)} \\
\quad \text{if (a}_i \geq \text{max)} \text{ then max} := a_i \\
\quad \text{if (a}_i \leq \text{min)} \text{ then min} := a_i \\
\text{max: the largest integer in the seq., min: the smallest integer in the sequence.}

\text{Description:}

\times \text{Set the maximum and the minimum to the first integer in the sequence.}

\times \text{Compare the max and the min with the next element in the seq.} \\
\quad \text{if max < this element, set the max to be that element.} \\
\quad \text{Also, if the min > element, set the min to be that element.}

\times \text{Repeat the previous step if there are more integers in the sequence.}

\times \text{Stop when there are no integers in the sequence.}
9) Linear Search

1, 3, 4, 5, 6, 8, 9, 11

n = 8, x = 7

\[ i \leq n \implies x \neq a_i \]

\[ i = 1 \]
\[ i = 2 \]
\[ i = 3 \]
\[ i = 4 \]
\[ i = 5 \]
\[ i = 6 \]
\[ i = 7 \]
\[ i = 8 \]
\[ i = 9 \]

\[ i = 9 > 8 \]

\[ \text{location} = 9 \implies \text{7 is not found.} \]
b) Binary Search

\[ i, j, 3, 4, 5, 6, 8, 9, 11 \]

\[ i = 1, j = 8, x = 7 \]

\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

\[ i, j, 3, 4, 5, 6, 8, 9, 11 \]

\[ a_i = 1, \quad a_j = 8 \]

\[ m = \left\lfloor \frac{9}{2} \right\rfloor = 4 \]

\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

\[ i, j, 3, 4, 5, 6, 8, 9, 11 \]

\[ 7 > 5 \quad (T) \]

\[ i = m + 1 = 5 \]

\[ 5 \quad 6 \quad 7 \quad 8 \]

\[ 6, 8, 9, 11 \]

\[ a_i = a_m \]

\[ i < j \quad (5 < 8) \]

\[ m = \left\lfloor \frac{13}{2} \right\rfloor = 6 \]

\[ 7 > 8 \quad (F) \]

\[ i = j = 6 \]
\[ a_i \neq a_j \]

\[ c < j \ (c < 6) \]

\[ m = \left\lceil \frac{11}{2} \right\rceil = 5 \]

\[ 7 > 6 \ (T) \]

\[ c \leq 6 \]

\[ 6 < 6 \ (F) \]

\[ 7 = a(i) = a(6) = 8 \ (F) \]

in locations 0, it isn't found.
<table>
<thead>
<tr>
<th></th>
<th>1-</th>
<th></th>
<th>2-</th>
<th></th>
<th>3-</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Discrete Disc.
Discrete Disc.

Eng. Haneen

\[ n = 6 \]

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
6 & 2 & 3 & 1 & 5 & 4
\end{array}
\]

\[ j = 2 \]

\[ a_i < a_j \text{ (F)} \]

\[ j - i - 1 = 2 - 1 - 1 = 0 \]

\[ a_2 = a_1 \]

\[ a_1 = m \]

\[ m + 6 = 2 \]

\[ a = 2 \]

\[ a = 6 \]

\[ a = 1 \]

\[ a = 4 \]

\[ a = 5 \]
2 - j = 3
2 6 3 1 5 4

aj > ai \Rightarrow i = i + 1 = 2

2 6 3 1 5 4

aj > ai (F)

j - i - 1 = 3 - 2 - 1 = 0.
a_2 = a_2, \quad a_2 = m _{s3}

2 6 3 1 5 4

2 3 6 1 5 4
Discrete Disc.

Eng. Haneen

\[ j = y \]
\[ a_j = a_i \ (E) \]

\[ j - i - 1 = y - 1 - 1 = 2 \]

\[ a_4 = a_3 \]
\[ a_3 = a_2 \]
\[ a_2 = a_1 \]

\[ a_i = m = 1 \]

\[ 1 \ 2 \ 3 \ 6 \ 5 \ 4 \]
4. \( j = 5 \)

\[
\begin{align*}
&\text{If } j > a_i \text{ (T)} \\
&\text{If } c = 2 \\
&\text{If } j > a_i \text{ (T)} \\
&\text{If } c = 3 \\
&\text{If } j > a_i \text{ (T)} \\
&\text{If } c = 4 \\
&\text{If } j > a_i \text{ (F)}.
\end{align*}
\]

5. \( j = 6 \)

\[
\begin{align*}
&123654 \\
&\text{While } \\
&\text{123564} \\
&123456
\end{align*}
\]
a) \( f(x) = 17x + 11 \).

For \( x > 1 \):

\[
17x + 11 \leq 17x + 11 x = 28x
\]

\[
17x + 11 \leq 28x
\]

Then \( 17x + 11 = O(x) \) s.t. \( k = 1, c = 28 \)

\[
\therefore x > 1
\]

\[
\therefore x \leq x^2
\]

\[
\therefore x = O(x^2)
\]

\[
\therefore 17x + 11 \leq 28x^2
\]

\[
\therefore 17x + 11 = O(x^2) \quad k = 1, c = 28.
\]
b) \( P(x) = x^2 + 1000 \)

\[
x^2 + 1000 \leq x^2 + 1000x^2 = 1001x^2
\]

\[\Rightarrow x^2 + 1000 = O(x^2), \quad k = 1, \quad c = 1001\]

c) \( P(x) = x \log x \)

For \( x \geq 1 \)

\[
\log x < x \quad \Rightarrow x \log x < x^2
\]

\[\Rightarrow x \log x = O(x^2), \quad k = 1, \quad c = 1\]
d) \( f(x) = x^{n/2} \).

For all \( x \): No \( k \) s.t.
\[
\frac{x^n}{2} \leq cx^2 \quad x > k.
\]

\[\therefore f(x) \text{ isn't } o(x^2).\]

e) \( f(x) = 2^x \).

No \( k \) achieves that:
\[
2^x \leq cx^2 \quad x > k.
\]

\[\therefore f(x) \text{ isn't } o(x^2).\]
9) $P(\infty) = \left\lfloor x \right\rfloor \left\lceil x \right\rceil$

$\left\lfloor x \right\rfloor \leq x$

$\therefore \left\lfloor x \right\rfloor \text{ is } O(1)$

$\left\lceil x \right\rceil \leq x + 1 \leq x + x \quad x \geq 1$

$\left\lceil x \right\rceil \leq 2x \quad x \geq 1$

$\therefore \left\lceil x \right\rceil = O(x) \quad c \leq 2, k \geq 1$

$\therefore \left\lfloor x \right\rfloor \cdot \left\lceil x \right\rceil = O(x \cdot x) = O(x^2) \quad c \leq 2, k \geq 1$

$2^x + 17 \text{ is } O(3^x)$.

For $x > 3$

$17 \leq 3^x \Rightarrow 2^x < 3^x$.

$2^x + 17 \leq 3^x + 3^x = 2(3^x)$.

$\therefore 2^x + 17 \text{ is } O(3^x) \quad k = 3, c \leq 2$
(n^2 + n^2 \log n)(\log n + 1) + (17\log n + 19)(n^3 + 2).

\[ f_1 = (n^2 + n^2 \log n)(\log n + 1) \]
\[ g_2 = n^3 + n^2 \log n \]
\[ O(n^3) \quad O(n^2 \log n) \]
\[ g_1 = O(\max(n^3, n^2 \log n)) = O(n^3) \]

\[ g_2 = \log n + 1 \]
\[ O(\log n) \quad O(1) \]
\[ g_2 = O(\max(\log n, 1)) = O(\log n) \]

\[ f_2 = (17 \log n + 19)(n^3 + 2). \]
\[ O(\log n) + O(1) + o(n^3) + o(1) \]
\[ O(\log n) \quad O(n^3). \]
\[ O(n^3 \log n). \]
\[ P(x) \begin{array}{c} \Rightarrow 26.9) \\
\end{array} \]

\[ P(x) \begin{array}{c} \leq O(\max(n^3 \log n, n^3 \log n)) = O(n^3 \log n) \end{array} \]

b) \((2^n + n^3) (n^2 + 3^n)\)

\[ \begin{array}{c}
\frac{\alpha(2^n)}{\alpha(n^2)} + \frac{\alpha(n^3)}{\alpha(3^n)} \\
\frac{\alpha(2^n)}{\alpha(n^2)} + \frac{\alpha(n^3)}{\alpha(3^n)} \\
\end{array} \]

\[ \begin{array}{c}
O(2^n) \cdot O(3^n) \\
O(2^n \times 3^n) = O(6^n) \\
\end{array} \]

c) \((n^n + n \cdot 2^n + 3^n) (n! + s^n)\)

\[ \begin{array}{c}
\frac{\alpha(n^n)}{\alpha(n^n)} + \frac{\alpha(2^n)}{\alpha(2^n)} + \frac{\alpha(n!)}{\alpha(s^n)} \\
\frac{\alpha(n^n)}{\alpha(n^n)} + \frac{\alpha(2^n)}{\alpha(2^n)} + \frac{\alpha(n!)}{\alpha(s^n)} \\
\end{array} \]

\[ \begin{array}{c}
O(n^n) \\
O(n^n \times n!) \\
\end{array} \]
9) \(3x + 7 \leq x\).

To prove that \(3x + 7 \in \Theta(x)\), we want to prove:

1. \(3x + 7 = O(x)\).
2. \(x = O(3x + 7)\).

1. For \(x > 1\)
   \[3x + 7 \leq 3x + 7x = 10x\]
   Then \(3x + 7 = O(x)\) \(k=1, c=10\).

2. For \(x > 1\)
   \[x \leq 3x + 7\]
   \[\therefore x = O(3x + 7)\] \(k=1, c=1\).

Then \(3x + 7 = \Theta(x)\).
\(d) \log(x^2 + 1), \log_2 x\)

1. \(\log(x^2 + 1) = O(\log x)\).

For \(x > 1\)

\[x^2 + 1 \leq x^2 + x^2 = 2x^2\]

\[\log(x^2 + 1) \leq \log(2x^2) = 2\log(2x)\]

\[2\log(2x) = 2\log 2 + 2\log x\]

\[\therefore 2\log(2x) = O(\log x)\]

\[\therefore \log(x^2 + 1) = O(\log x)\]
2- \( \log(x) = O(\log(x^2 + 1)) \).

For \( x > 1 \):
\[
x \leq x^2 + 1
\]
\[
\log x \leq \log x^2 + 1
\]
\[
\therefore \log(x) = O(\log(x^2 + 1))
\]

Then \( \log(x^2 + 1) = \Theta \log(x) \).