Mechanical Vibrations

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EXAMPLE 1.9

A hinged rigid bar of length $l$ is connected by two springs of stiffnesses $k_1$ and $k_2$ and is subjected to a force $F$ as shown in Fig. 1.33(a). Assuming that the angular displacement of the bar ($\theta$) is small, find the equivalent spring constant of the system that relates the applied force $F$ to the resulting displacement $x$. 

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displacements $l_1 \sin \theta$, $l_2 \sin \theta$, and $l \sin \theta$,

$$x_1 = l_1 \theta, \ x_2 = l_2 \theta \ \text{and} \ x = l \theta,$$

$$k_1 x_1(l_1) + k_2 x_2(l_2) = F(l)$$

or

$$F = k_1 \left( \frac{x_1 l_1}{l} \right) + k_2 \left( \frac{x_2 l_2}{l} \right)$$

By expressing $F$ as $k_{eq} x$, Eq. (E.1) can be written as

$$F = k_{eq} x = k_1 \left( \frac{x_1 l_1}{l} \right) + k_2 \left( \frac{x_2 l_2}{l} \right)$$

Using $x_1 = l_1 \theta$, $x_2 = l_2 \theta$, and $x = l \theta$, Eq. (E.2) yields the desired result:

$$k_{eq} = k_1 \left( \frac{l_1}{l} \right)^2 + k_2 \left( \frac{l_2}{l} \right)^2$$
\[ T = mg(l \sin \theta) \]
\[ T = mgl\theta \]
\[ T = k_t\theta \]
\[ k_t = mgl \]
1. *Equivalent translational mass.* The kinetic energy of the two masses is given by

\[ T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 \]
EXAMPLE 1.11

Find the equivalent mass of the system shown in Fig. 1.38, where the rigid link 1 is attached to the pulley and rotates with it.
Solution: Assuming small displacements, the equivalent mass \( m_{eq} \) can be determined using the equivalence of the kinetic energies of the two systems. When the mass \( m \) is displaced by a distance \( x \), the pulley and the rigid link 1 rotate by an angle \( \theta_p = \theta_1 = x/r_p \). This causes the rigid link 2 and the cylinder to be displaced by a distance \( x_2 = \theta_2 l_1 = xl_1/r_p \). Since the cylinder rolls without slippage, it rotates by an angle \( \theta_c = x_2/r_c = xl_1/r_p r_c \). The kinetic energy of the system \( (T) \) can be expressed (for small displacements) as:

\[
T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_p \dot{\theta}_p^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} J_c \dot{\theta}_c^2 + \frac{1}{2} m_c \dot{x}_2^2
\]

\[
T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_p \left( \frac{\dot{x}}{r_p} \right)^2 + \frac{1}{2} \left( \frac{m_1 l_1^2}{3} \right) \left( \frac{\dot{x}}{r_p} \right)^2 + \frac{1}{2} m_2 \left( \frac{\dot{x}}{r_p} \right)^2
\]

\[
+ \frac{1}{2} \left( \frac{m_c r_c^2}{2} \right) \left( \frac{\dot{x} l_1}{r_p r_c} \right)^2 + \frac{1}{2} m_c \left( \frac{\dot{x} l_1}{r_p} \right)^2
\]

\[
T = \frac{1}{2} m_{eq} \dot{x}^2
\]

we obtain the equivalent mass of the system as

\[
m_{eq} = m + \frac{J_p}{r_p^2} + \frac{1}{3} \frac{m_1 l_1^2}{r_p^2} + \frac{m_2 l_1^2}{r_p^2} + \frac{1}{2} \frac{m_c l_1^2}{r_p^2} + m_c \frac{l_1^2}{r_p^2}
\]
Cam-Follower Mechanism

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