Chapter 2

Analysis of Statically Determinate Structures
Idealized Structure

• In real sense exact analysis of a structure can never be carried out.

• Estimates have always to be made of the loadings and strength of materials.

• Furthermore, points of application for the loadings must be estimated.

• Models or idealization should be made.
Support connection

- *Pin support and pin connection*
• *Fixed support & fixed connection*

- Fixed support
- Fixed-connected joint

- Typical "fixed-supported" connection (metal)
- Typical "fixed-supported" connection (concrete)
• **Roller support**

  typical “roller-supported” connection (concrete)
torsional spring support  

torsional spring joint

(c)
Support for coplanar structures

<table>
<thead>
<tr>
<th>Type of Connection</th>
<th>Idealized Symbol</th>
<th>Reaction</th>
<th>Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>light cable</td>
<td><img src="image" alt="Light Cable" /></td>
<td>$F$</td>
<td>One unknown. The reaction is a force that acts in the direction of the cable or link.</td>
</tr>
<tr>
<td>weightless link</td>
<td><img src="image" alt="Weightless Link" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rollers</td>
<td><img src="image" alt="Rollers" /></td>
<td>$F$</td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>rocker</td>
<td><img src="image" alt="Rocker" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>smooth pin or hinge</td>
<td><img src="image" alt="Smooth Pin or Hinge" /></td>
<td>$F_x$, $F_y$</td>
<td>Two unknowns. The reactions are two force components.</td>
</tr>
<tr>
<td>fixed support</td>
<td><img src="image" alt="Fixed Support" /></td>
<td>$F_x$, $F_y$, $M$</td>
<td>Three unknowns. The reactions are the moment and the two force components.</td>
</tr>
</tbody>
</table>

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(a) Idealized framing plan
fixed-connected beam

fixed-connected overhanging beam

idealized beam

idealized beam
Chapter 2

idealized framing plan
concrete slab is reinforced in two directions, poured on plane forms

idealized framing plan for one-way slab action requires $L_2/L_1 \geq 2$
The floor of a classroom is supported by the bar joists shown in Fig. 2–15a. Each joist is 15 ft long and they are spaced 2.5 ft on centers. The floor itself is made from lightweight concrete that is 4 in. thick. Neglect the weight of the joists and the corrugated metal deck, and determine the load that acts along each joist.

**Solution**
The dead load on the floor is due to the weight of the concrete slab. From Table 1–3 for 4 in. of lightweight concrete it is \((4)(8 \text{ lb/ft}^2) = 32 \text{ lb/ft}^2\). From Table 1–4, the live load for a classroom is \(40 \text{ lb/ft}^2\). Thus the total floor load is \(32 \text{ lb/ft}^2 + 40 \text{ lb/ft}^2 = 72 \text{ lb/ft}^2\). For the floor system, \(L_1 = 2.5 \text{ ft}\) and \(L_2 = 15 \text{ ft}\). Since \(L_2/L_1 > 1.5\) the concrete slab is treated as a one-way slab. The tributary area for each joist is shown in Fig. 2–15b. Therefore the uniform load along its length is

\[ w = 72 \text{ lb/ft}^2(2.5 \text{ ft}) = 180 \text{ lb/ft} \]

This loading and the end reactions on each joist are shown in Fig. 2–15c.
The flat roof of the steel-frame building in Fig. 2–16a is intended to support a total load of 2 kN/m$^2$ over its surface. If the span of beams $AD$ and $BC$ is 5 m and the space between them ($AB$ and $DC$) is 3 m, determine the roof load within region $ABCD$ that is transmitted to beam $BC$. 
Solution
In this case $L_1 = 5 \text{ m}$ and $L_2 = 4 \text{ m}$. Since $L_2/L_1 = 1.25 < 1.5$, we have two-way slab action. The tributary loading is shown in Fig. 2–16b, where the shaded trapezoidal area of loading is transmitted to member $BC$. The peak intensity of this loading is $(2 \text{ kN/m}^2)(2 \text{ m}) = 4 \text{ kN/m}$. As a result, the distribution of load along $BC$ is shown in Fig. 2–16b. This process of tributary load transmission should also be calculated for the two square regions to the right of $BC$ in Fig. 2–16a, and this additional load should then be placed on $BC$.
Principle of superposition

• The total displacement or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings caused by each of the external loads acting separately.

• Linear relationships among loads, stresses and displacements
Superposition requirements

• Material must be behave in a linear elastic manner.

• The geometry of the structure must not undergo significant change. Small displacement theory applies.
Equation of Equilibrium

- In x-y plane

\[ \sum F_x = 0 \]
\[ \sum F_y = 0 \]
\[ \sum M_o = 0 \]

Internal loading
Determinacy & Stability

- **Determinacy**: when all the forces in structure can be determined from equilibrium equation, the structure is referred to as *statically determinate*. Structure having more unknown forces than available equilibrium equations called *statically indeterminate*.

- If $N$ is number of structure parts & $r$ is number of unknown forces:
  
  $r = 3n$, statically determinate

  $r > 3n$, statically indeterminate
Classify determinate & indeterminate structure

\( r = 3 \)
\( n = 1 \)
\( 3 = 3(1) \)  \( \text{Statically Determinate} \)
$r = 5$
$n = 1$
$5 > 3(1) \quad \text{Statically indeterminate}$
$2^{\text{nd}} \text{ degree}$

$r = 6$
$n = 2$
$\sigma = 3(2) \quad \text{Statically determinate}^{\text{Chapter 2}}$
$r = 10$
$n = 3$
$10 > 3(3) \text{ Statically indeterminate}$

$1^{st}$ degree

$r = 7$
$n = 2$
$7 > 3(2) \text{ Statically indeterminate, } 1^{st}$ degree
\[ r = 10 \]
\[ n = 2 \]
\[ 10 > 3(2) \quad \text{Statically indeterminate} \]
\[ 4^{th} \text{ degree} \]
• Stability

Partial Constraints

\[ \sum F_x = 0 \ 	ext{will not be satisfied} \Rightarrow \text{unstable with this loading case} \]
• **Improper Constraints**

This can occur if all the support reactions are concurrent at a point.

\[ P \times d \neq 0 \]
• This can occur also when the reactive forces are all parallel

In General

$r < 3n$, Then the structure is **Unstable**

$r \geq 3n$, Also, **Unstable** if member reactions are concurrent or parallel or some of the components form a collapsible mechanism
Classify the structure Stable or Unstable

\[ r = 3 \]
\[ n = 1 \]
\[ 3 = 3(1) \quad \text{no special cases} \quad \Rightarrow \text{Sable} \]
\( r = 8 \)
\( n = 2 \)

\( 8 > 3(2) \) \( \text{no special cases} \) \( \Rightarrow \) Sable

\( r = 3 \)
\( n = 1 \)

\( 3 = 3(1) \) \( \Rightarrow \) Unstable
\[ r = 7 \]
\[ n = 3 \]
\[ 7 < 3(3) \implies \text{Unstable} \]
\( r = 9, \ n = 2, \ 9 > 6, \)

Statically indeterminate to the third degree

\textit{Ans.}

\( r = 9, \ n = 1, \ 9 > 3, \)

Statically indeterminate to the sixth degree

\textit{Ans.}

(This frame has no closed loops.)
Application of Equilibrium Equation
Application of Equation of Equilibrium

Procedure steps

1. If not given, establish a suitable $x$ - $y$ coordinate system.

2. Draw a free body diagram (FBD) of the object under analysis.

3. Apply the three equations of equilibrium to solve for the unknowns.
Example 1

Determine the Reactions
\[ \sum F_x = 0 \implies A_x - 60 \cos 60 = 0 \]

\[ A_x = 30k \]

\[ \sum M_A = 0 \implies -60 \sin 60 \cdot 10 + 60 \cos 60 \cdot 1 + B_y \cdot 14 - 50 = 0 \]

\[ B_y = 38.5k \]

\[ \sum F_y = 0 \implies -60 \sin 60 + 38.5 + A_y = 0 \]

\[ A_y = 13.4k \]
Example 2

Determine the Reactions
\[ \sum F_x = 0 \quad \Rightarrow A_x = 0 \]
\[ \sum F_y = 0 \quad \Rightarrow A_x - 60 - 60 = 0 \]
\[ A_x = 120kN \]
\[ \sum M_A = 0 \quad \Rightarrow -60(4) - 60(6) + M_A = 0 \]
\[ M_A = 600kN \]
Example 3

Determine the Reactions
\[
\sum M_A = 0 \implies -3500(3.5) + \frac{4}{5} N_B (4) + \frac{3}{5} N_B (10) = 0
\]
\[
N_B = 1331.5 \, Ib
\]
\[
\sum F_x = 0 \implies A_x - \frac{4}{5} (1331.5) = 0
\]
\[
A_x = 1070 \, Ib
\]
\[
\sum F_y = 0 \implies A_y - 3500 + \frac{3}{5} (1331.5) = 0
\]
\[
A_y = 2700 \, Ib
\]
Example 4

Determine the Reactions
\[ \sum M_{B \text{ (Right)}} = 0 \Rightarrow 15C_y - 6000 = 0 \]

\[ C_y = 400 \text{lb} \]

\[ \sum M_{\text{at point } A} = 0 \Rightarrow 400(35) - 6000 + 8000(10) + M_A = 0 \]

\[ M_A = 72000 \text{lb}\cdot\text{ft} \]

\[ \sum F_x = 0 \Rightarrow A_x = 0 \]

\[ \sum F_y = 0 \Rightarrow A_y + 400 - 8000 = 0 \]

\[ A_y = 7600 \text{lb} \]

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Example 4

Determine the Reactions
\[ \sum M_{B(Right)} = 0 \Rightarrow 2C_y - 6(1) = 0 \]

\[ C_y = 3 \text{kN} \]

\[ \sum M_A = 0 \Rightarrow C_x (1.5) + 3(4) - 6(3) - 8(2) = 0 \]

\[ C_x = 14.7 \text{kN} \]

\[ \sum F_x = 0 \Rightarrow A_x - 14.7 - 8\left(\frac{3}{5}\right) = 0 \]

\[ A_x = 9.87 \text{kN} \]

\[ \sum F_y = 0 \Rightarrow A_y + 3 - 6 - 8\left(\frac{4}{5}\right) = 0 \]

\[ A_y = 9.4 \text{kN} \]
Example 5

Determine the Reactions
\[
\sum M_{B_{(Left)}} = 0 \Rightarrow -6A_y + 8(3) = 0
\]

\[A_y = 4kN\]

\[
\sum M_{D_{(Left)}} = 0 \Rightarrow -3C_y - 4(11) + 8(8) = 0
\]

\[C_y = 6.67kN\]

\[
\sum F_x = 0 \Rightarrow E_x = 0
\]

\[
\sum F_y = 0 \Rightarrow E_y - 8 - 8 + 4 + 6.67 = 0
\]

\[E_y = 5.33kN\]

\[
\sum M_{D_{(right)}} = 0 \Rightarrow -8(2) + 5.33(4) - M_E = 0
\]

\[M_E = 5.33kN.m\]
The side girder shown in the photo supports the boat and deck. An idealized model of this girder is shown in Fig. 2–31a, where it can be assumed A is a roller and B is a pin. Using a local code the anticipated deck loading transmitted to the girder is 6 kN/m. Wind exerts a \textit{resultant} horizontal force of 4 kN as shown, and the mass of the boat that is supported by the girder is 23 Mg. The boat’s mass center is at G. Determine the reactions at the supports.
Solution

**Free-Body Diagram.** Here we will consider the boat and girder as a single system, Fig. 2–31b. As shown, the distributed loading has been replaced by its resultant.

**Equations of Equilibrium.** Applying Eqs. 2–2 in sequence, using previously calculated results, we have

\[ \pm \Sigma F_x = 0; \quad 4 - B_x = 0 \]

\[ B_x = 4 \text{ kN} \quad \text{(Ans.}) \]

\[ \downarrow \Sigma M_B = 0; \quad 22.8(1.90) - A_y(2) + 225.6(5.40) - 4(0.30) = 0 \]

\[ A_y = 630.2 \text{ kN} = 630 \text{ kN} \quad \text{(Ans.)} \]

\[ + \uparrow \Sigma F_y = 0; \quad 630.2 - B_y - 22.8 - 225.6 = 0 \]

\[ B_y = 382 \text{ kN} \quad \text{(Ans.)} \]

*Note:* If the girder alone had been considered for this analysis then the normal forces at the shoes C and D would have to first be calculated using a free-body diagram of the boat. (These forces exist if the cable pulls the boat snug against them.) Equal but opposite normal forces along with the cable force at E would then act on the girder when its free-body diagram is considered. The same results would have been obtained; however, by considering the boat-girder system, these normal forces and the cable force become internal and do not have to be considered.
Example 6

Determine the Reactions
\[ \sum M_A = 0 \Rightarrow C_y (6) - 180(1.5) - 60(1.5) - 254.6 \cos 45(4.5) \\
- 254.6 \sin 45(1.5) - 84.9 \cos 45(4.5) + 84.9 \cos 45(4.5) = 0 \]

\[ C_y = 240 kN \]

\[ \sum M_B = 0 \Rightarrow -C_x (6) + 240(3) + 60(4.5) + 84.9 \left( \frac{3\sqrt{2}}{2} \right) = 0 \]

\[ C_x = 195 kN \]

\[ \sum F_x = 0 \Rightarrow -A_x - 195 + 180 + 60 + 254.6 \cos 45 + 84.9 \cos 45 = 0 \]

\[ A_x = 285 kN \]

\[ \sum F_y = 0 \Rightarrow -A_y + 240 - 254.6 \sin 45 + 84.9 \sin 45 = 0 \]

\[ A_y = 120 kN \]
Problem 1

Determine the Reactions
Problem 2

Determine the Reactions
Problem 3

Determine the Reactions