Chapter 3

Trusses
Trusses in Building

[Diagram showing the components of a truss structure, including roof, purlins, top cord, gusset plates, bottom cord, knee brace, span, and bay.]
Trusses Types for Building

(a) scissors

(b) Howe

(c) Pratt

(d) fan
Truss Bridge
Truss Bridge Types

Pratt (a)

Howe (b)

Warren (with verticals) (c)
Classification of coplanar Truss

Simple Truss
Compound Truss
Complex Truss
Determinacy

For plane truss

If \( b + r = 2j \) \hspace{1cm} \text{statically determinate}
If \( b + r > 2j \) \hspace{1cm} \text{statically indeterminate}

Where
\( b = \text{number of bars} \)
\( r = \text{number of external support reaction} \)
\( j = \text{number of joints} \)
Stability

For plane truss

If \( b + r < 2j \) unstable

The truss can be statically determinate or indeterminate \((b + r \geq 2j)\) but unstable in the following cases:

External Stability: truss is externally unstable if all of its reactions are concurrent or parallel.
Internal stability:

Internally stable

Internally unstable

Internally unstable
Classify each of the truss as stable, unstable, statically determinate or indeterminate.

\( b = 19 \quad \ldots \quad r = 3 \quad \ldots \quad j = 11 \)
\( b + r = 22 \)
\( 2j = 2(11) = 22 \)
Statically determinate & Stable

\( b = 15 \quad \ldots \quad r = 4 \quad \ldots \quad j = 9 \)
\( b + r = 19 \)
\( 2j = 2(9) = 18 \)
Statically indeterminate & Stable
\( b = 9 \quad \ldots \quad r = 3 \quad \ldots \quad j = 6 \)
\[ b + r = 12 \]
\[ 2j = 2(6) = 12 \]
Statically determinate & Stable

\( b = 12 \quad \ldots \quad r = 3 \quad \ldots \quad j = 8 \)
\[ b + r = 15 \]
\[ 2j = 2(8) = 16 \]
Unstable
Stability of Compound Truss
The Method of Joints

**STEPS FOR ANALYSIS**

1. If the *support reactions* are not given, draw a *FBD* of the entire truss and determine all the support reactions using the equations of equilibrium.

2. Draw the *free-body diagram of a joint* with one or two unknowns. *Assume that all unknown member forces act in tension (pulling the pin)* unless you can determine by inspection that the forces are compression loads.

3. Apply the scalar equations of equilibrium, \( \sum F_X = 0 \) and \( \sum F_Y = 0 \), to determine the unknown(s). If the answer is *positive*, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).

4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.
The Method of Joints

**Diagrams:**

1. A structure with two bars meeting at joint B. Bar AB is 2 m long with a 45° angle at B. Bar BC is 2 m long. A force of 500 N acts at joint B.

2. Force diagrams:
   - **F_{BA}** (tension): A force acts from B to A.
   - **F_{BC}** (compression): A force acts from B to C.
   - **F_{BA}** (tension): A force acts from B to A at 45°.
   - **F_{BC}** (compression): A force acts from B to C at 45°.
Example 1

Solve the following truss
\[ \sum F_y = 0; \]
\[ 4 + F_{AG} \sin 30 = 0 \Rightarrow F_{AG} = -8\,KN \quad C \]
\[ \sum F_x = 0; \]
\[ F_{AB} - 8 \cos 30 = 0 \Rightarrow F_{AB} = 6.93\,KN \quad T \]
\[ \sum F_y = 0; \]
\[ -F_{GB} - 3 \cos 30 = 0 \Rightarrow F_{GB} = -2.6 \text{KN} \quad \text{C} \]
\[ \sum F_x = 0; \]
\[ 8 - 3 \sin 30 + F_{GF} = 0 \Rightarrow F_{GF} = -6.50 \text{KN} \quad \text{C} \]

\[ \sum F_y = 0; \]
\[ F_{BF} \sin 60 - 2.6 \sin 60 = 0 \Rightarrow F_{BF} = 2.6 \text{KN} \quad \text{T} \]
\[ \sum F_x = 0; \]
\[ F_{BC} + 2.6 \cos 60 + 2.6 \cos 60 - 6.93 = 0 \]
\[ \Rightarrow F_{BC} = 4.33 \text{KN} \quad \text{T} \]
**Group Work**

Determine the force in each member
Zero Force Member

1- If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero-force members.

\[
\sum F_x = 0; F_{CB} = 0
\]

\[
\sum F_y = 0; F_{CD} = 0
\]
2- If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member.
Example 2

Find Zero force member of the following truss.
Method of Section
The Method of Section
The Method of Section

**STEPS FOR ANALYSIS**

1. Decide how you need to “cut” the truss. This is based on:
   a) where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).

2. Decide which side of the cut truss will be easier to work with (minimize the number of reactions you have to find).

3. **If required**, determine the necessary support reactions by drawing the FBD of the entire truss and applying the EofE.
4. Draw the FBD of the selected part of the cut truss. We need to indicate the unknown forces at the cut members. Initially we assume all the members are in tension, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member must be in compression. (Please note that you can also assume forces to be either tension or compression by inspection as was done in the previous example above.)

5. Apply the equations of equilibrium (EofE) to the selected cut section of the truss to solve for the unknown member forces. Please note that in most cases it is possible to write one equation to solve for one unknown directly.
Example 2

Solve the CF & GC members in the truss

\[
\begin{align*}
A_x &= 0 \\
A_y &= 639.7 \text{ lb} \\
E_y &= 639.7 \text{ lb}
\end{align*}
\]
\[ \sum M_E = 0; \]
\[ F_{CF} \sin 30(12) + 300(6.93) = 0 \Rightarrow F_{CF} = -346.4\text{lb} \quad \text{C} \]
\[ \sum M_A = 0; \]
\[ F_{GC}(12) - 300(6.93) - 346.4 \sin 30(12) = 0 \Rightarrow F_{GC} = 346.4 \text{lb} \text{ T} \]
Example 3

Solve the GF & GD members in the truss
\[ \sum M_D = 0 \rightarrow F_{GF} \cos 26.6(3) + 7(3) = 0 \Rightarrow F_{GF} = -7.83kN \quad C \]

\[ \sum M_o = 0 \rightarrow -F_{GD} \sin 56.3(6) - 7(3) + 2(6) = 0 \Rightarrow F_{GF} = -1.8kN \quad C \]
Example 4

Solve the ED & EB members in the truss
Example 5

Solve the BC & MC members in the K-truss
\[ \sum M_L = 0 \rightarrow -2900(15) + F_{BC}(20) = 0 \Rightarrow F_{BC} = 2175 \text{lb} \quad \text{T} \]

At Joint B \[ \sum F_y = 0 \rightarrow F_{MB} = 1200 \text{lb} \quad \text{T} \]

For the total cutting part \[ \sum F_y = 0 \rightarrow 2900 - 1200 + 1200 - F_{ML} \Rightarrow F_{ML} = 2900 \text{lb} \quad \text{T} \]
At Joint M

\[ \sum F_x = 0 \rightarrow \frac{3}{\sqrt{13}} F_{MC} + \frac{3}{\sqrt{13}} F_{MK} = 0 \]

\[ \sum F_y = 0 \rightarrow 2900 - 1200 - \frac{2}{\sqrt{13}} F_{MC} + \frac{2}{\sqrt{13}} F_{MK} = 0 \]

\[ F_{MK} = 1532 Ib \quad C \]

\[ F_{MC} = 1532 Ib \quad T \]
Group work 1

Solve All members
Group work 2

Solve All members
Group work.

Solve members CH, CI
Example 4___Compound Truss

Solve the truss
\[ \sum M_C = 0 \rightarrow -5(4) + 4(2) + F_{HG}(4 \sin 60) = 0 \Rightarrow F_{HG} = 3.46 \text{kN} \]
Joint A ⇒ $F_{AB}$ & $F_{AI}$
Joint H ⇒ $F_{HI}$ & $F_{HJ}$
Joint I ⇒ $F_{IJ}$ & $F_{IB}$
Joint B ⇒ $F_{BC}$ & $F_{BJ}$
Joint J ⇒ $F_{JC}$
Example 5—Compound Truss

A_x = 0
A_y = 3 k
3 k
F_y = 3 k

6 ft
6 ft
6 ft
6 ft
6 ft
6 ft

45°
45°
45°

C
H
G

D
B
E

F

12 ft
\[ + \sum M_B = 0; \quad -3(6) - F_{DG}(6 \sin 45^\circ) + F_{CE} \cos 45^\circ (12) + F_{CE} \sin 45^\circ (6) = 0 \quad (1) \]
\[ + \sum F_y = 0; \quad 3 - 3 - F_{BH} \sin 45^\circ + F_{CE} \sin 45^\circ = 0 \quad (2) \]
\[ \pm \sum F_x = 0; \quad -F_{BH} \cos 45^\circ + F_{DG} - F_{CE} \cos 45^\circ = 0 \quad (3) \]

From Eq. (2), \( F_{BH} = F_{CE} \); then solving Eqs. (1) and (3) simultaneously yields

\[ F_{BH} = F_{CE} = 2.68 \text{ k (C)} \quad F_{DG} = 3.78 \text{ k (T)} \]
Analysis of each connected simple truss can now be performed using the method of joints. For example, from Fig. 3–30c, this can be done in the following sequence.

*Joint A:* Determine the force in $AB$ and $AD$.
*Joint D:* Determine the force in $DC$ and $DB$.
*Joint C:* Determine the force in $CB$. 

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**Diagram:**
- Nodes A, B, C, and D connected by bars.
- Forces: 3 kN at A, 2.68 kN at C, 2.68 kN at D, 3.78 kN at B.
- Angles: $45^\circ$ at C, $45^\circ$ at D, $45^\circ$ at A, $45^\circ$ at B.
Example 5 __ Compound Truss

Solve the truss
Joint A ⇒ $F_{AE}$ & $F_{AB}$
Joint B ⇒ $F_{EB}$

After Solve $F_{AE}$
Joint A ⇒ $F_{AF}$ & $F_{AG}$
Complex Truss

\[ S_i = S'_i + xS_i \]

\[ S_{EC} = S'_{EC} + xS_{EC} = 0 \]

\[ x = ? \]
Complex Trusses

\[ r + b = 2j, \]
\[ \begin{array}{ccc}
3 & 9 & 2(6) \\
\end{array} \]

Determinate • Stable •

\[ S_{AD} \]

\[ S_{EC} + x s_{EC} = 0 \]

\[ x = \frac{S_{EC}'}{s_{EC}} \]

\[ F_{AD} = \]

\[ S_i = S_i' + x s_i \]
\[ S_{EC} = S'_{EC} + xS_{EC} = 0 \]
\[ x = ? \]

\[ S_i = S'_i + xS_i \]

<table>
<thead>
<tr>
<th>Member</th>
<th>( S'_i )</th>
<th>( S_i )</th>
<th>( xS_i )</th>
<th>( S_i )</th>
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<tbody>
<tr>
<td>AB</td>
<td></td>
<td></td>
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<tr>
<td>AC</td>
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<tr>
<td>EC</td>
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Example 5_Complex Truss
**Superposition.** We require

\[ S_{DB} = S'_{DB} + xs_{DB} = 0 \]

Substituting the data for \( S'_{DB} \) and \( s_{DB} \), where \( S'_{DB} \) is negative since the force is compressive, we have

\[-2.50 + x(1.167) = 0 \quad x = 2.143\]
The values of $xs_i$ are recorded in column 4 of Table 1, and the actual member forces $S_i = S'_i + xs_i$ are listed in column 5.

<table>
<thead>
<tr>
<th>Member</th>
<th>$S'_i$</th>
<th>$s_i$</th>
<th>$xs_i$</th>
<th>$S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CB$</td>
<td>3.54</td>
<td>-0.707</td>
<td>-1.52</td>
<td>2.02 (T)</td>
</tr>
<tr>
<td>$CD$</td>
<td>-3.54</td>
<td>-0.707</td>
<td>-1.52</td>
<td>5.05 (C)</td>
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<tr>
<td>$FA$</td>
<td>0</td>
<td>0.833</td>
<td>1.79</td>
<td>1.79 (T)</td>
</tr>
<tr>
<td>$FE$</td>
<td>0</td>
<td>0.833</td>
<td>1.79</td>
<td>1.79 (T)</td>
</tr>
<tr>
<td>$EB$</td>
<td>0</td>
<td>-0.712</td>
<td>-1.53</td>
<td>1.53 (C)</td>
</tr>
<tr>
<td>$ED$</td>
<td>-4.38</td>
<td>-0.250</td>
<td>-0.536</td>
<td>4.91 (C)</td>
</tr>
<tr>
<td>$DA$</td>
<td>5.34</td>
<td>-0.712</td>
<td>-1.53</td>
<td>3.81 (T)</td>
</tr>
<tr>
<td>$DB$</td>
<td>-2.50</td>
<td>1.167</td>
<td>2.50</td>
<td>0</td>
</tr>
<tr>
<td>$BA$</td>
<td>2.50</td>
<td>-0.250</td>
<td>-0.536</td>
<td>1.96 (T)</td>
</tr>
</tbody>
</table>
Space Truss

- Determinacy and Stability

\[ b + r < 3 \]

unstable truss

\[ b + r = 3j \]

statically determinate-check stability

\[ b + r > 3j \]

statically indeterminate-check stability
Space Truss

- x, y, z, Force Components

\[ l = \sqrt{x^2 + y^2 + z^2} \]
\[ F_x = F\left(\frac{x}{l}\right) \]
\[ F_y = F\left(\frac{y}{l}\right) \]
\[ F_z = F\left(\frac{z}{l}\right) \]
\[ F = \sqrt{F_x^2 + F_y^2 + F_z^2} \]
### Support Reactions

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<td><strong>Roller</strong></td>
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<tr>
<td><img src="image" alt="Slotted Roller" /></td>
<td><img src="image" alt="Slotted Roller" /></td>
</tr>
<tr>
<td><strong>Slotted Roller Constrained in a Cylinder</strong></td>
<td><strong>Slotted Roller Constrained in a Cylinder</strong></td>
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<td><img src="image" alt="Ball-and-Socket" /></td>
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<td><strong>Ball-and-Socket</strong></td>
<td><strong>Ball-and-Socket</strong></td>
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</tbody>
</table>
1- If all but one of the members connected to a joint lie on the same plane, and provided no external load act on the joint, then the member not lying in the plane of the other members must subjected to zero force.

\[ \Sigma F_z = 0, \quad F_D = 0 \]
Zero Force Member

2- If it has been determined that all but two of several members connected at a joint support zero force, then the two remaining members must also support zero force, provided they don’t lie along the same line and no external load act on the joint.

\[ \Sigma F_z = 0, \quad F_B = 0 \]
\[ \Sigma F_y = 0, \quad F_D = 0 \]
Example 6__Space Truss
\[ \sum M_y = 0; \quad -600(4) + B_x(8) = 0 \quad B_x = 300 \text{ lb} \]
\[ \sum M_z = 0; \quad C_y = 0 \]
\[ \sum M_x = 0; \quad B_y(8) - 600(8) = 0 \quad B_y = 600 \text{ lb} \]
\[ \sum F_x = 0; \quad 300 - A_x = 0 \quad A_x = 300 \text{ lb} \]
\[ \sum F_y = 0; \quad A_y - 600 = 0 \quad A_y = 600 \text{ lb} \]
\[ \sum F_z = 0; \quad A_z - 600 = 0 \quad A_z = 600 \text{ lb} \]
\[ \Sigma F_y = 0; \quad -600 + F_{BE}\left(\frac{8}{12}\right) = 0 \quad F_{BE} = 900 \text{ lb (T)} \]
\[ \Sigma F_x = 0; \quad 300 - F_{BC} - 900\left(\frac{4}{12}\right) = 0 \quad F_{BC} = 0 \]
\[ \Sigma F_z = 0; \quad F_{BA} - 900\left(\frac{8}{12}\right) = 0 \quad F_{BA} = 600 \text{ lb (C)} \]
\[ \sum F_z = 0; \quad 600 - 600 + F_{AC} \sin 45^\circ = 0 \]
\[ F_{AC} = 0 \]

\[ \sum F_y = 0; \quad -F_{AE} \left( \frac{2}{\sqrt{5}} \right) + 600 = 0 \]
\[ F_{AE} = 670.8 \text{ lb (C)} \]

\[ \sum F_x = 0; \quad -300 + F_{AD} + 670.8 \left( \frac{1}{\sqrt{5}} \right) = 0 \]
\[ F_{AD} = 0 \]
\[ \sum F_x = 0; \]
\[ \sum F_z = 0; \]
\[ F_{DE} = 0 \]
\[ F_{DC} = 0 \]

**Joint C.** By observation of the free-body diagram, Fig. 3–38f,

\[ F_{CE} = 0 \]
Example 7__Space Truss
Solution

The free-body diagram, Fig. 3–39a, indicates there are eight unknown reactions for which only six equations of equilibrium are available for solution. Although this is the case, the reactions can be determined, since \( b + r = 3 j \) or \( 16 + 8 = 3(8) \).

To spot the zero-force members, we must compare the conditions of joint geometry and loading to those of Figs. 3–36 and 3–37. Consider joint \( F \), Fig. 3–39b. Since members \( FC, FD, FE \) lie in the \( x-y \) plane and \( FG \) is not in this plane, \( FG \) is a zero-force member. (\( \Sigma F_y = 0 \) must be satisfied.) In the same manner, from joint \( E \), Fig. 3–39c, \( FE \) is a zero-force member, since it does not lie in the \( y-z \) plane. (\( \Sigma F_x = 0 \) must be satisfied.) Returning to joint \( F \), Fig. 3–39b, it can be seen that \( F_{FD} = F_{FC} = 0 \), since \( F_{FE} = F_{FG} = 0 \), and there are no external forces acting on the joint.

The numerical force analysis of the joints can now proceed by analyzing joint \( G \) (\( F_{GF} = 0 \)) to determine the forces in \( GH, GB, GC \). Then analyze joint \( H \) to determine the forces in \( HE, HB, HA \); joint \( E \) to determine the forces in \( EA, ED \); joint \( A \) to determine the forces in \( AB, A \), and \( A \); joint \( B \) to determine the force in \( BC \) and \( B_x, B_z \); joint \( D \) to determine the force in \( DC \) and \( D_y, D_z \); and finally, joint \( C \) to determine \( C_x, C_y, C_z \).