Structure Analysis I
Chapter 6
Influence Line for Statically Determinate Structures
Influence lines

- Influence lines provide a systematic procedure of how force in a given part of structure varies as the applied loads moves along the structure.
Procedure for Analysis

• Place a **unit load** at various locations, \( x \) along the member, and at each location use static to determine the value of the function (Reaction, Shear, Moment) at the specified point.

• If the IL for a vertical force reaction at a point on a beam is to be constructed, consider the reaction to be positive at the point when it acts upward on the beam.

• If a shear or moment IL is to be drawn for a point, take the shear or moment at the point as positive according to the same sign convention used for drawing shear and moment diagram.
• All statically determinate beams will have IL that consist of straight line segments. After some practice one should be able to minimize computations and locate the unit load only at points representing the end points of each line segment.
Example 1

Draw the IL for the Reaction at A

\[ \sum M_B = 0; \quad -A_y (10) + 1 (7.5) = 0 \]

\[ A_y = 0.75 \]

\[ \sum \sum M_B = 0; \quad A_y (10) + 1 (5) = 0 \]

\[ A_y = 0.5 \]
IL Equation

\[ \sum M_B = 0 \]

\[-A_y(10) + (10 - x)(1) = 0 \]

\[ A_y = 1 - \frac{1}{10} x \]
Example 2

Draw the IL for the Reaction at B

<table>
<thead>
<tr>
<th>$x$</th>
<th>$B_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>7.5</td>
<td>1.5</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

Influence line for $B_y$

$B_y = \frac{1}{5} x$
Example 3

Draw the IL for the Shear at C
Load from A to C

Load > C
influence line for $V_C$

$V_C = 1 - \frac{1}{10} x$

$V_C = -\frac{1}{10} x$
Example 4

Draw the IL for the Moment at C
Load from A to C

4 m < x ≤ 12 m

Load > C

0 ≤ x < 4 m
Example 1

Draw the IL for the Shear & Moment at B
Example 2

Draw the IL for the Shear & Moment at C
Example 3

Internal Hinge Example
Determine the shear & moment at point D
Example 3-Continue

Determine the maximum positive moment that can be developed at point \( D \) in the beam shown in Fig. 6–19a due to a concentrated moving load of 4000 lb, a uniform moving load of 300 lb/ft, and a beam weight of 200 lb/ft.
\[ M_D = 500 \left[ \frac{1}{2} (25 - 10)(3.33) \right] + 4000(3.33) - 200 \left[ \frac{1}{2} (10)(3.33) \right] \]
\[ = 22500 \text{ lb} \cdot \text{ft} = 22.5 \text{ k} \cdot \text{ft} \]
Example 4

Determine the moment at C in the two cases
Case 1

\[ M_c = 6(1.25) + 10(2.5) = 32.5 \text{ kN.m} \]
Case 2

\[ M_c = 6(2.5) + 10(1.25) = 27.5 \text{ kN.m} \]
Qualitative Influence Lines
deflected shape

influence line for $V_C$
Example 5

Sketch the influence line for the vertical reaction at A
deflected shape

influence line for $A_y$
Example 6

Sketch the influence line for the shear at B
deflected shape

influence line for $V_B$
Example 6-b

Sketch the influence line for the moment at B
Influence Line for Floor Girders
Example 7

Draw the influence line for the shear at panel CD
1 at \( x = 0 \)

\[ A_y = 1 \quad B_y = 0 \]

\[ \Sigma M_G = 0; \quad F_y = 0.333 \]

\[ \Sigma F_y = 0; \quad V_{CD} = 0.333 \]
at $x = 20$ ft

$B_y = 0$

$C_y = 1$

$\sum M_G = 0; F_y = 0.333$

$\sum F_y = 0; V_{CD} = -0.333$

$G_y$

$F_y$

$10$ ft

$20$ ft

$M$

$V_{CD}$

$F_y = 0.333$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$V_{CD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.333</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>-0.333</td>
</tr>
<tr>
<td>30</td>
<td>0.333</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>
influence line for $V_{CD}$
Example 8

Draw the influence line for the moment at F
1 at x = 2 m

$\Sigma M_A = 0; \; B_y = 0.5$

$\Sigma M_H = 0; \; G_y = 0.0714$

$\Sigma M_F = 0; \; M_F = 0.429$

$G_y = 0.0714$

<table>
<thead>
<tr>
<th>x</th>
<th>$M_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.429</td>
</tr>
<tr>
<td>4</td>
<td>0.857</td>
</tr>
<tr>
<td>8</td>
<td>2.571</td>
</tr>
<tr>
<td>10</td>
<td>2.429</td>
</tr>
<tr>
<td>12</td>
<td>2.286</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
</tr>
</tbody>
</table>
influence line for $M_F$
Example 9

Truss Example

Determine The force in member GB
\[ F_{GB} \cos 45 = R_E \]

\[ F_{GB} = 1.41R_E \]

for the second part

\[ F_{GB} \cos 45 = -R_A \]

\[ F_{GB} = -1.41R_A \]
Example 10

Truss Example
Determine the force in member GF, BF
\[ \sum M_B = 0 \Rightarrow 40R_D = -F_{GF} \ (17.3) \]
\[ F_{GF} = -2.3R_D \]
\[ F_{BF} \cos 30 = R_D \rightarrow F_{BF} = 1.15R_D \]

\[ \sum M_F = 0 \Rightarrow 20R_A = -F_{GF} \ (17.3) \]
\[ F_{GF} = -1.15R_A \]
\[ F_{BF} \cos 30 = -R_A \rightarrow F_{BF} = -1.15R_A \]
Example 11

Truss Example
Determine The force in member CG

<table>
<thead>
<tr>
<th>( x )</th>
<th>( F_{GC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
</tr>
</tbody>
</table>
Example 11

Determine the largest force that can be developed in member $B$ bridge truss shown in Fig. 6–26a due to a moving force of 20 moving distributed load of 0.6 k/ft. The loading is applied at the t
\[ \sum M_H = 0; \quad -F_{BC} (15) + 0.25 (40) = 0 \]

\[ F_{BC} = 0.667 \text{ (T)} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( F_{BC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0.667</td>
</tr>
<tr>
<td>40</td>
<td>1.33</td>
</tr>
<tr>
<td>60</td>
<td>0.667</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
</tr>
</tbody>
</table>

Influence line for \( F_{BC} \)
**Influence Line.** A plot of the tabular values yields the influence line, Fig. 6–26(d). By inspection, BC is a primary member. Why?

**Concentrated Live Force.** The largest force in member BC occurs when the moving force of 20 k is placed at \( x = 40 \) ft. Thus,

\[
F_{BC} = (1.33)(20) = 26.7 \text{ k}
\]

**Distributed Live Load.** The uniform live load must be placed over the entire deck of the truss to create the largest tensile force in BC.* Thus,

\[
F_{BC} = \left[ \frac{1}{2}(80)(1.33) \right]0.6 = 32.0 \text{ k}
\]

**Total Maximum Force**

\[
(F_{BC})_{\text{max}} = 26.7 \text{ k} + 32.0 \text{ k} = 58.7 \text{ k}
\]

*The largest tensile force in member GB of Example 6–15 is created when the distributed load acts on the deck of the truss from \( x = 0 \) to \( x = 8 \) m, Fig. 6–24(d).
IL due to Series of Concentrated Loads

\[
( V_C )_1 = 1(0.75) + 4(0.625) + 4(0.5) = 5.25k
\]
\[
(V_C)_2 = 1(-0.125) + 4(0.75) + 4(0.625) = 5.375k
\]
\( (V_C)_3 = 1(0) + 4(-1.25) + 4(0.75) = 2.5k \)
\[
(M_C)_1 = 2(7.5) + 4(6.5) + 3(5.0) = 56.0 \text{ k}\cdot\text{ft}
\]
\[
(M_C)_2 = 2(4.5) + 4(7.5) + 3(6.0) = 57.0 \text{ k}\cdot\text{ft}
\]
\[
(M_C)_3 = 2(0) + 4(3.0) + 3(7.5) = 34.5 \text{ k}\cdot\text{ft}
\]
Example 12

Determine the maximum positive shear created at point $B$ in the beam shown in Fig. 6–30$a$ due to the wheel loads of the moving truck.

Solution

The influence line for the shear at $B$ is shown in Fig. 6–30$b$. 

\[ V_B \]

influence line for $V_B$

(b)
\((V_B)_{\text{max}} = 4(-0.05) + 9(-0.2) + 15(0.5) + 10(0.2)\)

\[ = 7.5 \text{ k} \]
Example 13

\[
\left( M_B \right)_{\text{max}} = 8(1.2) + 3(0.4) = 10.8 \text{ kN} \cdot \text{m}
\]
Example 14

Determine the maximum compressive force developed in member $BG$ of the side truss in Fig. 6–32a due to the right side wheel loads of the car and trailer. Assume the loads are applied directly to the truss and move only to the right.

![Diagram of the truss with loads and forces]
1.5-kN Load at Point C. In this case

\[ F_{BG} = 1.5 \text{kN} (-0.625) + 4(0) + 2 \text{kN} \left( \frac{0.3125}{3 \text{ m}} \right)(1 \text{ m}) \]

\[ = -0.729 \text{ kN} \]

4-kN Load at Point C. By inspection this would seem a more reasonable case than the previous one.

\[ F_{BG} = 4 \text{kN} (-0.625) + 1.5 \text{kN} \left( \frac{-0.625}{6 \text{ m}} \right)(4 \text{ m}) + 2 \text{kN}(0.3125) \]

\[ = -2.50 \text{ kN} \]

2-kN Load at Point C. In this case all loads will create a compressive force in BC.

\[ F_{BG} = 2 \text{kN} (-0.625) + 4 \text{kN} \left( \frac{-0.625}{6 \text{ m}} \right)(3 \text{ m}) + 1.5 \text{kN} \left( \frac{-0.625}{6 \text{ m}} \right)(1 \text{ m}) \]

\[ = -2.66 \text{ kN} \quad \text{Ans.} \]

Since this final case results in the largest answer, the critical loading occurs when the 2-kN load is at C.
Absolute Maximum Shear and Moment

\[ V_{\text{abs max}} \]

\[ M_{\text{abs max}} \]
The location of Maximum Moment is at

\[ x = \frac{\bar{x}}{2} \]
Example 15

Determine the absolute maximum moment in the simply supported bridge deck shown in Fig. 6–37a.
**Solution**

The magnitude and position of the resultant force of the system are determined first, Fig. 6–37a. We have

\[ \Sigma F_r = F_R; \quad F_R = 2 + 1.5 + 1 = 4.5 \text{ k} \]

\[ \Sigma M_{BC} = 4.5\bar{x} - 1.5(10) + 1(15) \]

\[ \bar{x} = \frac{6.67}{6} \text{ ft} \]

Let us first assume the absolute maximum moment occurs under the 1.5-k load. The load and the resultant force are positioned equidistant from the beam's centerline, Fig. 6–37b. Calculating \( A_y \) first, Fig. 6–37b, we have

\[ \Sigma M_B = 0; \quad -A_y(30) + 4.5(16.67) = 0 \quad A_y = 2.50 \text{ k} \]

Now using the left section of the beam, Fig. 6–37c, yields

\[ \Sigma M_S = 0; \quad -2.50(16.67) + 2(10) + M_S = 0 \]

\[ M_S = 21.7 \text{ k-ft} \]
There is a possibility that the absolute maximum moment may occur under the 2-k load, since $2 \text{k} > 1.5 \text{k}$ and $F_R$ is between both 2 k and 1.5 k. To investigate this case, the 2-k load and $F_R$ are positioned equidistant from the beam’s centerline, Fig. 6–37d. Show that $A_y = 1.75 \text{k}$ as indicated in Fig. 6–37e and that

$$M_S = 20.4 \text{k} \cdot \text{ft}$$

By comparison, the absolute maximum moment is

$$M_S = 21.7 \text{k} \cdot \text{ft} \quad \text{Ans.}$$

which occurs under the 1.5-k load, when the loads are positioned on the beam as shown in Fig. 6–37b.