Chapter Two

Force Vectors
APPLICATION OF VECTOR ADDITION

There are four concurrent cable forces acting on the bracket.

How do you determine the resultant force acting on the bracket?
## 2.1 Scalars and Vectors

<table>
<thead>
<tr>
<th></th>
<th>Scalars</th>
<th>Vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td>mass, volume</td>
<td>force, velocity</td>
</tr>
<tr>
<td><strong>Characteristics</strong></td>
<td>It has a magnitude</td>
<td>It has a magnitude</td>
</tr>
<tr>
<td></td>
<td>(positive or negative)</td>
<td>and direction</td>
</tr>
<tr>
<td><strong>Addition rule</strong></td>
<td>Simple arithmetic</td>
<td>Parallelogram law</td>
</tr>
<tr>
<td><strong>Special Notation</strong></td>
<td>None</td>
<td>Bold font, a line, an</td>
</tr>
<tr>
<td></td>
<td></td>
<td>arrow or a “carrot”</td>
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</tbody>
</table>
2.2 VECTOR OPERATIONS

Multiplication and Division of a vector by a scalar
Vector Addition and Subtraction

Parallelogram Law:

Triangle method (always ‘tip to tail’):

Vector subtraction $\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$
Sine law:
\[ \frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \]

Cosine law:
\[ C = \sqrt{A^2 + B^2 - 2AB \cos c} \]
“Resolution” of a vector is breaking up a vector into components. It is kind of like using the parallelogram law in reverse.
2.3 Vector Addition of Forces
2.4 Addition of a system of coplanar forces

**Scalar Notation.** The rectangular components of force $\mathbf{F}$ shown in Fig. 2–15a are found using the parallelogram law, so that $\mathbf{F} = F_x + F_y$. Because these components form a right triangle, their magnitudes can be determined from

$$F_x = F \cos \theta \quad \text{and} \quad F_y = F \sin \theta$$
VECTOR NOTATION

• We ‘resolve’ vectors into components using the x and y axes system.

• Each component of the vector is shown as a magnitude and a direction.

• The directions are based on the x and y axes. We use the “unit vectors” $\mathbf{i}$ and $\mathbf{j}$ to designate the x and y axes.
For example,

\[ \mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad \text{or} \quad \mathbf{F}' = F'_x \mathbf{i} + F'_y \mathbf{j} \]

The x and y axes are always perpendicular to each other. Together, they can be directed at any inclination.
ADDITION OF SEVERAL VECTORS

- Step 1 is to resolve each force into its components.
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.
Example of this process,

\[
F_R = F_1 + F_2 + F_3 \\
= F_{1x}i + F_{1y}j - F_{2x}i + F_{2y}j + F_{3x}i - F_{3y}j \\
= (F_{1x} - F_{2x} + F_{3x})i + (F_{1y} + F_{2y} - F_{3y})j \\
= (F_{Rx})i + (F_{Ry})j
\]
You can also represent a 2-D vector with a magnitude and angle.

\[ F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} \]

\[ \theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) \]
EXAMPLE

Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force.

Plan:

a) Resolve the forces in their x-y components.
b) Add the respective components to get the resultant vector.
c) Find magnitude and angle from the resultant components.
EXAMPLE (continued)

\[ F_1 = \{ 15 \sin 40^\circ \mathbf{i} + 15 \cos 40^\circ \mathbf{j} \} \text{ kN} \]
\[ = \{ 9.642 \mathbf{i} + 11.49 \mathbf{j} \} \text{ kN} \]

\[ F_2 = \{ -(12/13)26 \mathbf{i} + (5/13)26 \mathbf{j} \} \text{ kN} \]
\[ = \{ -24 \mathbf{i} + 10 \mathbf{j} \} \text{ kN} \]

\[ F_3 = \{ 36 \cos 30^\circ \mathbf{i} - 36 \sin 30^\circ \mathbf{j} \} \text{ kN} \]
\[ = \{ 31.18 \mathbf{i} - 18 \mathbf{j} \} \text{ kN} \]
EXAMPLE
(continued)
Summing up all the $i$ and $j$ components respectively, we get,

$$F_R = \{ (9.642 - 24 + 31.18) \, i + (11.49 + 10 - 18) \, j \} \, \text{kN}$$

$$= \{ 16.82 \, i + 3.49 \, j \} \, \text{kN}$$

$$F_R = ((16.82)^2 + (3.49)^2)^{1/2} = 17.2 \, \text{kN}$$

$$\phi = \tan^{-1}(3.49/16.82) = 11.7^\circ$$
\[ F_1 = \{ (4/5) \times 850 \ i - (3/5) \times 850 \ j \} \ N \]
\[ = \{ 680 \ i - 510 \ j \} \ N \]
\[ F_2 = \{ -625 \sin(30^\circ) \ i - 625 \cos(30^\circ) \ j \} \ N \]
\[ = \{ -312.5 \ i - 541.3 \ j \} \ N \]
\[ F_3 = \{ -750 \sin(45^\circ) \ i + 750 \cos(45^\circ) \ j \} \ N \]
\[ \{ -530.3 \ i + 530.3 \ j \} \ N \]
GROUP PROBLEM SOLVING (continued)

Summing up all the $i$ and $j$ components respectively, we get,

\[ F_R = \{ (680 - 312.5 - 530.3) \, i + (-510 - 541.3 + 530.3) \, j \} \, N \]

\[ = \{ -162.8 \, i - 521 \, j \} \, N \]

\[ F_R = ((162.8)^2 + (521)^2)^{\frac{1}{2}} = 546 \, N \]

\[ \phi = \tan^{-1}(521/162.8) = 72.64^\circ \quad \text{or} \]

From Positive x axis \( \theta = 180 + 72.64 = 253^\circ \)