DSP DISCUSSION

CHAPTER ONE

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**Signal:** is a function which is dependent of some variables which are independent variables.

Signal: $f(x_1, x_2, \ldots)$, $x$ : time, distance, temperature, ……

In DSP the independent variable is number (small $n$): $f(n)$

**System:** defined as a physical device that performs an operation on a signal.

**Processed the signal:** occurs when we pass the signal through the system.
Basic element of DSP system
Advantages of Digital over analog signal processing

1) Flexibility: simply changing program.
2) Amenable to full integration
3) More accuracy
4) Less sensitivity to components (R,L,C) and environment changes
5) Dynamic range
6) Easy adjustment of processor characteristic by changing coefficients (numbers)
   In analog we change value of R,L,C.
7) Easy storage in Taps, disks,.....

Disadvantages of Digital

1) Increase complexity (A/D, D/A)
2) It has certain limitations due to A/D (fs=2fo for no aliasing)
3) Power dissipation.
**Classification of Signals:**

(1) **Mono channel versus Multichannel**

- multichannel generated by multisource
- Ex: Electrocardiograms (ECG)

(2) **One dimensional versus multidimensional**

- If the signal is a function of one independent variable then it is one-dimensional signal otherwise it is multidimensional.
- Ex: - Any picture is 2-dim signal, \( f(x,y) \)
  - Black and white TV picture is a 3-dim signal, \( f(x,y,t) \)
  - for color TV: the signal is 3-dim \( (x,y,t) \) and 3-channel \( (R,G,B) \)

\[
I(x,y,t) = \begin{bmatrix}
I_r(x,y,t) \\
I_g(x,y,t) \\
I_b(x,y,t)
\end{bmatrix}
\]
(3) Real valued or complex value signal

S1(t) = A \sin \pi t \rightarrow \text{real signal}
S2(t) = A e^{j\pi t} \rightarrow \text{complex signal}

(4) Continuous time versus discrete time signal

(a) CT + CV \rightarrow \text{analog}
(b) DT + CV \rightarrow \text{discrete}
(c) DT + DV \rightarrow \text{digital}

(5) Deterministic versus random

deterministic signal// can be described by mathematical equation.
The concept of frequency in CT and DT signals

Continuous-time sinusoidal signal

\[ x(t) = A \cos(\Omega t + \theta) \]

A: amplitude, \( \theta \): phase

\( \Omega \): analog frequency in rad

\( \Omega = 2\pi F \), \( F = \frac{1}{T} \), T: period

Discrete-time sinusoidal signal

\[ x(n) = A \cos(wn + \theta) \]

n: sampler number, \( w = 2\pi f \)

\( f = \frac{1}{N} \), N: period

\( x(t) \Leftrightarrow x(n) \)

\( \Omega \Leftrightarrow w \)

\( F \Leftrightarrow f \)

\( T \Leftrightarrow N \)
For Continuous-time sinusoidal signal:

1) $x_a(t)$ always periodic.

2) Continuous-time sinusoidal signals with different frequencies are themselves distinct.

3) Increasing the frequency $F$ results in an increase in the rate of oscillation of the signal.
(1) Periodic of discrete-sinusoidal signal

\( x(n) \) is periodic only if its frequency in Hertz is rational number.

\[
f = \frac{\text{integer}}{\text{integer}} = \frac{k}{N}
\]

**proof:**

\( x(n) = A \cos(wn + \theta) \) .................. (1)

\( x(n + N) = A \cos(w(n + N) + \theta) = A \cos(wn + wN + \theta) \)

\( = A \cos(wn + \theta + 2\pi fN) \) .................. (2)

for: (1)=(2)

\( 2\pi fN = 2\pi k \quad k = \pm 1, \pm 2, \ldots \)

\( f = \frac{k}{N} \)

Note: analog sinusoidal signal always periodic.
Determine which of the following sinusoids are periodic and compute their fundamental period:

(a) \( \cos(0.01\pi n) \)

\[
\omega = 0.01\pi = 2\pi f \quad \Rightarrow \quad f = \frac{0.01\pi}{2\pi} = \frac{1}{200} = \frac{k}{N} \quad (\text{rational})
\]

\( \Rightarrow \) periodic with period \( N = 200 \)
(b) $\cos(30\pi n/105)$

\[
\omega = \frac{30\pi}{105} = 2\pi f \quad \Rightarrow f = \frac{30\pi}{2\pi (105)} = \frac{1}{7} = \frac{k}{N} \quad (\text{rational})
\]

$\Rightarrow$ periodic with period $N=7$
(c) $\sin(3n)$

$w = 3 = 2\pi f \rightarrow f = \frac{3}{2\pi}$ (not rational)

$\Rightarrow$ non – periodic
\[ x(n) = \cos\left(\frac{\pi n}{6}\right) \cos\left(\frac{\pi n}{3}\right) \]

\[
\cos(x) \cos(y) = \frac{1}{2} \left[ \cos(x + y) + \cos(x - y) \right]
\]

\[
x(n) = \cos\left(\frac{\pi n}{3}\right) \cos\left(\frac{\pi n}{6}\right) = \frac{1}{2} \left[ \cos\left(\frac{\pi n}{3} + \frac{\pi n}{6}\right) + \cos\left(\frac{\pi n}{3} - \frac{\pi n}{6}\right) \right] = \frac{1}{2} \left[ \cos\left(\frac{3\pi n}{6}\right) + \cos\left(\frac{\pi n}{6}\right) \right]
\]

\[
\cos\left(\frac{3\pi n}{6}\right) \Rightarrow w = \frac{3\pi}{6} = 2\pi f \quad \rightarrow f = \frac{3}{12} = \frac{1}{4} \quad \text{(rational, } N_1 = 4)\]

\[
\cos\left(\frac{\pi n}{6}\right) \Rightarrow w = \frac{\pi}{6} = 2\pi f \quad \rightarrow f = \frac{1}{12} \quad \text{(rational, } N_2 = 12)\]

\[ \rightarrow x(n) \text{ periodic with period} = \text{LCM}(N_1, N_2) = \text{LCM}(4, 12) = 12 \]
Determine which of the following sinusoids are periodic and compute their fundamental period:

(a) \( x(t) = 3 \cos(5t + \pi/6) \)

analog sinusoidal signal \( \Rightarrow \) always periodic

\[ \Omega = 5 = 2\pi F \rightarrow F = \frac{5}{2\pi} \Rightarrow T \text{ (period)} = \frac{1}{F} = \frac{2\pi}{5} = 1.256 \]
(2) Discrete time sinusoid whose radian frequencies are separated by integer multiples of $2\pi$ are identical

**proof** :

$x_1(n) = A \cos(w_1 n + \theta)$ ........................ (1)

$x_2(n) = A \cos(w_2 n + \theta)$

$w_2 = w_1 + 2\pi k$

$x_2(n) = A \cos((w_1 + 2\pi k)n + \theta) = A \cos(w_1 n + 2\pi kn + \theta)$

$= A \cos(w_1 n + \theta)$ ...........................(2)

$\Rightarrow (1) = (2)$

$w = w_o + 2\pi k$

$-\pi \leq w_o \leq \pi \rightarrow -\pi \leq 2\pi f \leq \pi$

$-\frac{1}{2} \leq f \leq \frac{1}{2}$

*for example:*

$x(n) = \cos(w_o n)$
**Note:**

- The period of the sinusoidal decreases as the frequency increases.
- The rate of oscillation increase as the frequency increase.
- The sequences of any two sinusoidal with frequencies in the range : \(-\pi \leq w \leq \pi \) or \(-0.5 \leq f \leq 0.5 \) are different (unique).
Sequence that result from a sinusoid with a frequency $|w| > \pi$ is identical to a sequence obtained from a sinusoidal with frequency $-\pi \leq w \leq \pi$.

For continuous-time sinusoids result in distinct signals for $\Omega$ or $F$ in the range $-\infty < F < \infty$ or $-\infty < \Omega < \infty$.

$w=360$ aliased with $w=0$, $w=30$ aliased with $w=390$ ............
The frequency range for discrete-time sinusoids is finite with duration $2\pi$:

choose, $0 \leq w \leq 2\pi$ ($0 \leq f \leq 1$) or $-\pi \leq w \leq \pi$ ($-0.5 \leq f \leq 0.5$) $\Rightarrow$ fundamental range
Analog to digital conversion

A/D converter

Sampler → Quantizer → Coder

$x_d(t)$ → $x(n)$ → $x_q(n)$ → 01011...

Analog signal → Discrete-time signal → Quantized signal → Digital signal
(1) Sampling

Analog signal $x_a(t)$ is sampled at a rate $F_s = 1/T$ to produce a discrete-time signal $x(n) = x_a(nT)$. The diagram illustrates the process with a waveform $x_a(t)$ and its sampled version $x(n)$. The sampling interval is $T$, and the discrete-time samples are taken at multiples of $nT$. The sampled signal is shown at intervals of $T, 2T, ..., 9T, ..., t = nT$.
\[ x_a(t) = A \cos(\Omega t + \theta) \Rightarrow x(nT_s) = A \cos(\Omega nT_s + \theta) = x(n) \]

\[ x_a(t) \Rightarrow x(nT_s) = x(n) \]

\[ t = nT_s = \frac{n}{F_s} \]

\[ x(n) = A \cos(\Omega nT_s + \theta) = A \cos(2\pi Fn \frac{1}{F_s} + \theta) \]

\[ = A \cos(2\pi n \frac{F}{F_s} + \theta) = A \cos(2\pi nf + \theta) \]

\[ f = \frac{F}{F_s} = F * T_s \iff \omega = \Omega * T_s \]

\textit{digital} frequency = analog frequency \cdot \text{sampling time}

for continuous: \(-\infty < (F, \Omega) < \infty\)

for discrete sinusoidal signals: \(\begin{cases} 
-0.5 \leq f \leq 0.5 \\
-\pi \leq \omega \leq \pi
\end{cases}\)

The infinite analog frequency is mapping into finite digital frequency.
To determine the mapping of any (alias) frequency above \( F_s/2 \) into the equivalent frequency below \( F_s/2 \), we can use \( F_s/2 \) to reflect or fold the alias frequency to the fundamental range.
**Continuous -time signal**

\[ \Omega = 2\pi F \]

rad \quad Hz

sec

\[ -\infty < \Omega < \infty \]

\[ -\infty < F < \infty \]

\[ f = \frac{F}{F_s}, \omega = \Omega \cdot T_s \]

**Discrete -time signal**

\[ w = 2\pi f \]

rad \quad cycles

sample \quad sample

\[ -\pi \leq w \leq \pi \]

\[ -0.5 \leq f \leq 0.5 \]
Consider the following two analog sinusoidal signal signals:
\[ x_1(t) = \cos(2\pi 10t) \quad , \quad x_2(t) = \cos(2\pi 50t) \quad , \quad F_s = 40 \text{ Hz} \]

Find the corresponding discrete sequence:

\[ F_s = 40 \text{ Hz} \quad , \quad F_1 = 10 \text{ Hz} \quad , \quad F_2 = 50 \text{ Hz} \]

\[ x_{1}(n) = x_a(nT_s) = \cos(2\pi 10n \frac{1}{40}) = \cos(\frac{\pi}{2} n) \]

\[ x_{2}(n) = x_a(nT_s) = \cos(2\pi 50n \frac{1}{40}) = \cos(5 \frac{\pi}{2} n) = \cos(5 \frac{\pi}{2} n + 2\pi k) \]

\[ = \cos(5 \frac{\pi}{2} n - 2\pi) = \cos(\frac{\pi}{2} n) \]

\[ \Rightarrow x_{1}(n) = x_{2}(n) \]

\[ \text{but } x_{1}(t) = x_{2}(t) \]

\[ \text{comment : The frequency } F_2 = 50 \text{ Hz is an alias of } F_1 = 10 \text{ Hz at } F_s = 40 \text{ Hz} \]

\[ F_{\text{fold}} = \frac{F_s}{2} = 20 \text{ Hz} \quad , \quad f = \frac{F}{F_s} = \frac{20}{40} = 0.5 \text{ Hz} \]
Note: All sinusoidal with frequency

\[ F_k = F_0 + k F_s , k = \pm 1, \pm 2, \ldots \ldots \]

\[ F_{\text{aliased}} = F_{\text{base}} + k F_s \]

\[ f_{\text{aliased}} = f_{\text{base}} + k \]

Leads a unique signal if sampled at Fs

Ex:

If \( F_0 = 10 \text{ Hz} \) find the alias frequencies of \( F_0 \) if \( F_s = 40 \text{ Hz} \)

\[ F_1 = 10 + 1(40) = 50 \text{ Hz} \]
\[ F_2 = 10 + 2(40) = 90 \text{ Hz} \]
\[ F_3 = 10 + 3(40) = 130 \text{ Hz} \]

\[ F_{\text{fold}} = \frac{F_s}{2} = 20 \text{ Hz} \]

\[ f = \frac{F}{F_s} = \frac{20}{40} = 0.5 \text{ Hz} \]
\[ F_1 = 10 + 1 \times 40 = 50 \text{ Hz} \]

\[ f = \frac{F}{F_s} = \frac{50}{40} = 1.25, \quad f_{\text{base}} = 1.25 - 1 = 0.25 \text{ Hz} \]

\[ f = \frac{F}{F_s} = \frac{10}{40} = 0.25 \text{ Hz} \]

\[ F_k = F_o + k F_s \]

\[ F_{\text{aliased}} = F_{\text{base}} + k F_s \]

\[ f_{\text{aliased}} = f_{\text{base}} + k \]

\( k = \pm 1, \pm 2, \ldots \)
$F_s = 8000 \text{ Hz}$
An analog signal contains frequencies up to 10 kHz

(a) What range of sampling frequencies allow exact reconstruction of this signal from its samples?

\[ F_s \geq 2F_{\text{max}} = 2(10) = 20 \text{ kHz} \]

\[ \rightarrow F_s \geq 20 \text{ kHz} \]

(b) If \( F_s = 8 \text{ kHz} \). Examine what happens to the frequency \( F_1 = 5 \text{ kHz} \)?

\[ F_{\text{fold}} = \frac{F_s}{2} = 4 \text{ kHz} < 5 \text{ kHz} \]

or \( F_s = 8 \text{ kHz} < 2F_1 = 10 \text{ kHz} \)

\[ \Rightarrow \text{aliasing happens} \]

\[ F_{\text{base}} = F_{\text{aliased}} - kF_{\text{fold}} = 5 - (1)4 = 1 \text{ kHz} \]
An analog signal contains frequencies up to 100 Hz

(a) What is the Nyquist rate of this signal?

\[ F_s = 2F_{\text{max}} = 2(100) = 200 \text{ Hz} \]

(b) If \( F_s = 250 \) sample/sec, what is the highest frequency that can be represented uniquely at this sampling?

\[ F_{\text{fold}} = \frac{F_s}{2} = 125 \text{ Hz} \quad , \quad f_{\text{fold}} = 0.5 \]
An analog signal \( x(t) = \sin(480\pi t) + 3\sin(720\pi t) \) is sampled 600 times per second.

(a) What is the Nyquist sampling rate?

\[
F_1 = \frac{480\pi}{2\pi} = 240 \text{ Hz}, \quad F_2 = \frac{720\pi}{2\pi} = 360 \text{ Hz}
\]

\[
F_{s,N} = 2F_{\text{max}} = 2(360) = 720 \text{ Hz}
\]

(b) Determine the folding frequency.

\[
F_{\text{fold}} = \frac{F_s}{2} = \frac{600}{2} = 300 \text{ Hz}
\]

(c) Find \( x(n) \).

\[
x(n) = x_a(nT_s) = \sin(480\pi nT_s) + 3\sin(720\pi nT_s)
\]

\[
= \sin(480\pi n \frac{1}{600}) + 3\sin(720\pi n \frac{1}{600}) = \sin(0.8\pi n) + 3\sin(1.2\pi n)
\]

\[
= \sin(0.8\pi n) + 3\sin(-0.8\pi n) = \sin(0.8\pi n) - 3\sin(0.8\pi n)
\]

\[
= -2\sin(0.8\pi n)
\]
(d) If \( x(n) \) is pass through an ideal D/A converter, what is the reconstructed signal \( y(t) \).

\[
y(t) = x(tF_s) = x\left(\frac{t}{T_s}\right) = -2\sin(0.8\pi t \cdot 600) = -2\sin(480\pi t)
\]
Sampling x(t) → by multiplying x(t) by train of impulses

\[ x(n) = x(t)P(t) \]
\[ X_s(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(f - nf_s) \]

\[ = \ldots + \frac{1}{T} X(f + 2f_s) + \frac{1}{T} X(f + f_s) + \frac{1}{T} X(f_s) + \frac{1}{T} X(f - f_s) + \frac{1}{T} X(f - 2f_s) + \ldots \]
Anti-aliasing Filter

![Diagram showing the concept of anti-aliasing filters in signal processing. The diagram illustrates the relationship between the original spectrum, the folded spectrum, and the reconstructed spectrum after anti-aliasing filtering. It highlights the importance of filtering to prevent aliasing artifacts.](image-url)
Suppose that an analog signal is given as:

\[ X(t) = 5\cos(2\pi \times 1000t) \]

and is sampled at the rate of 8000 Hz

(a) Sketch the spectrum for the original signal

\[
\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2} [\delta(f + f_0) + \delta(f - f_0)]
\]

\[ 5\cos(2\pi \times 1000 \times t) \leftrightarrow 2.5[\delta(f + 1000) + \delta(f - 1000)] \]

(b) Sketch the spectrum for the sampled signal from 0 to 20 kHz

\[
X_s(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(f - nf_s)
\]

\[ = \ldots + \frac{1}{T}X(f + 2f_s) + \frac{1}{T}X(f + f_s) + \frac{1}{T}X(f_s) + \frac{1}{T}X(f - f_s) + \frac{1}{T}X(f - 2f_s) + \ldots \]
Notice that the spectrum of the sampled signal contains the images of the original spectrum that the images repeat at multiplies of the sampling frequency $f_s, 2f_s, 3f_s, \ldots$ (8 kHz, 16 kHz, 24 kHz, …). All images must be removed since they convey no additional information.
- Sampling converts the analogue signal into discrete value of samples.

- We need to encode each sample value in order to store it in $b$ bits memory location.
\( x(t) = (0.9)^t \), \( F_s = 1 \) sample/sec
\( x(n) = (0.9)^n \)

If \( b = 4 \) bits \( \Rightarrow L = 16 \) levels (choose \( L = 11 \))
\( \{0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1\} \), \( \text{step}=0.1 \)

<table>
<thead>
<tr>
<th>( X(n)=x(nT) )</th>
<th>( Xq(n) )</th>
<th>( Eq(n)= xq(n)-x(n) )</th>
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Q. \( \text{step} = \Delta = \frac{x_{\text{max}} - x_{\text{min}}}{L-1} = \frac{1-0}{11-1} = 0.1 \)
Quantization step $\Delta = \frac{x_{\text{max}} - x_{\text{min}}}{L - 1}$

Quantization error: $e_q(n) = x_q(n) - x(n)$

$-\frac{\Delta}{2} \leq \text{error} \leq \frac{\Delta}{2}$
Quantization of sinusoidal signal

Average power of sinusoidal signal:

\[ P_{\text{sig}} = \frac{1}{2\pi} \int_{0}^{2\pi} S^2(t) \, dt = \frac{1}{2\pi} \int_{0}^{2\pi} (A \sin \omega t)^2 \, dt = \frac{A^2}{2} \]

Average power of quantized signal:

\[ e_q(t) = \frac{\Delta}{2T} t \quad \text{for} \quad (-T \leq t \leq T) \]

\[ P_q = \frac{1}{2T} \int_{-T}^{T} (e_q(t))^2 \, dt = \frac{1}{2T} \int_{-T}^{T} \left( \frac{\Delta}{2T} t \right)^2 \, dt = \]

\[ = \frac{1}{2T} \left( \frac{\Delta}{2T} \right)^2 \int_{-T}^{T} t^2 \, dt = \frac{\Delta^2}{12} \]
Signal to quantization noise ratio

the signal to quantization noise ratio & #13;

\[ SQNR = \frac{P_{\text{sig}}}{P_q} = \frac{\left( \frac{A^2}{2} \right)}{\Delta^2} \]

\[ \Delta = \frac{x_{\text{max}} - x_{\text{min}}}{L} = \frac{A - (-A)}{L} = \frac{2A}{L} \]

\[ SQNR = \frac{P_{\text{sig}}}{P_q} = \frac{\left( \frac{A^2}{2} \right)}{\frac{2A^2}{12L^2}} = \frac{3}{2}L^2 \]
(a) What is the sampling frequency and folding frequency?

\[ X(t) = 3\cos(600\pi t) + 2\cos(1800\pi t) \]

Operated at 10,000 bits/sec, quantized into 1024 levels

\[ L = 2^b = 1024 \rightarrow \log_2 1024 = \log_2 2^b \rightarrow \frac{\log 1024}{\log 2} = b = 10 \text{ bits/sample} \]

\[ F_s = \frac{\text{sample}}{\text{sec}} = \frac{\text{sample}}{\text{bit}} \cdot \frac{\text{bit}}{\text{sec}} = \left(\frac{1}{10}\right)(10,000) = 1000 \text{ sample/sec} \]

\[ F_{\text{fold}} = \frac{F_s}{2} = 500 \text{ Hz} \]

(b) What is the Nyquist rate?

\[ F_1 = \frac{600\pi}{2\pi} = 300 \text{ Hz} \quad , \quad F_2 = \frac{1800\pi}{2\pi} = 900 \text{ Hz} \]

\[ F_{s,N} = 2F_{\text{max}} = 2(900) = 1800 \text{ Hz} \]
(c) Find \( x(n) \).

\[
x(n) = x_a(nT_s) = 3\cos(600\pi n \frac{1}{1000}) + 2\cos(1800\pi n \frac{1}{1000}) \\
= 3\cos(0.6\pi n) + 2\cos(1.8\pi n) \\
= 3\cos(0.6\pi n) + 2\cos(-0.2\pi n) \\
= 3\cos(0.6\pi n) + 2\cos(0.2\pi n)
\]

(d) What is the resolution.

\[
\Delta = \frac{x_{\text{max}} - x_{\text{min}}}{L - 1} = \frac{5 - (-5)}{1024 - 1} = \frac{10}{1023}
\]
\[ X(n) = 6.35 \cos(\pi n/10) \] is quantized with a resolution = 0.02
How many bits are required in the A/D converter?

\[ \Delta = \frac{x_{\text{max}} - x_{\text{min}}}{L - 1} = \frac{6.35 - (-6.35)}{L - 1} = 0.02 \implies L = 636 = 2^b \]
\[ b = \frac{\log_{10} 636}{\log_{10} 2} = 9.3 \approx 10 \text{ bits} \]
Determine the bit rate and the resolution in the sampling of a seismic signal with dynamic range of 1 volt if the sampling rate is \( F_s = 20 \text{ samples/s} \) and we use an 8-bit A/D converter? What is the maximum frequency that can be present in the resulting digital seismic signal?

(a) Bit rate: \( \text{bit rate} = \frac{\text{bit}}{\text{sec}} = \frac{\text{bit}}{\text{sample}} \times \frac{\text{sample}}{\text{sec}} = b \times F_s = (8)(20) = 160 \text{ bits/sec} \)

(b) \( \Delta = \frac{x_{\text{max}} - x_{\text{min}}}{L - 1} = \frac{1}{2^8 - 1} = \frac{1}{255} \)

(c) \( F_{\text{fold}} = \frac{20}{2} = 10 \text{ Hz} \)
Part A - From Textbook [70%]:

- Solve all the following problems: 1.3, 1.5, 1.11, 1.15

Part B - External Problem [30%]:

The input signal to a digital communication link is:

\[ x(t) = 5 \cos (2\pi F_1 t) + 3 \cos (2\pi F_2 t) \]

the link is operated at 64 kbps, with 0.064 resolution. If \( F_1 = 2 \) kHz and \( F_2 = 3 \) kHz,

a) Sketch the spectrum for the original signal.
b) What is the frequencies in the resulting discrete-time signal \( x(n) \)
c) Sketch the spectrum of the sampled signal up to 20 kHz.
d) Sketch the recovered analog signal spectrum if an ideal lowpass filter with a
cutoff frequency of 4 kHz is used to recover the original signal.
e) Write expression of the recovered signal in the time domain.
   Comment on your results.
f) Determine the signal to quantization noise ratio in dB.
g) Resolve all parts if \( F_2 = 5 \) kHz