Refrigeration cycle
Objectives

- Know basic of refrigeration
- Able to analyze the efficiency of refrigeration system

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Ideal Vapor-Compression Refrigeration Cycle
Actual Vapor-Compression Refrigeration Cycle
Cascade refrigeration systems
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Refrigeration cycle

- Refrigeration is the transfer of heat from a lower temperature region to a higher temperature region.

- Refrigeration cycle is the vapor-compression refrigeration cycle, where the refrigerant is vaporized and condenses alternately and is compressed in the vapor phase.
Refrigerator and Heat Pump

- Cyclic refrigeration device operating between two constant temperature reservoirs.
- In the Carnot cycle heat transfers take place at constant temperature.
- If our interest is the cooling load, the cycle is called the Carnot refrigerator.
- If our interest is the heat load, the cycle is called the Carnot heat pump.
Refrigerator & Heat pump

- **Coefficient of performance, COP**

  \[
  COP = \frac{\text{Desired output}}{\text{Require input}}
  \]

- **Refrigerator**: is used to maintain the refrigerated space at a low temperature by removing heat from it

  \[
  COP_R = \frac{\text{Cooling effect}}{\text{Work input}} = \frac{Q_L}{W_{\text{net,in}}}
  \]

- **Heat pump**: heat transfers from a low-temperature medium to a high temperature medium

  \[
  COP_{HP} = \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_H}{W_{\text{net,in}}}
  \]

  \[\text{COP}_{HP} = \text{COP}_R + 1\]
Carnot refrigerator or a Carnot heat pump

The reversed Carnot cycle is the most efficient refrigeration cycle operating between two specified temperature levels.

A refrigerator or heat pump that operates on the reversed Carnot cycle is called a Carnot refrigerator or a Carnot heat pump.

\[
COP = \frac{\text{Desired output}}{\text{Require input}}
\]

\[
COP_{R,\text{Carnot}} = \frac{T_L (s_2 - s_1)}{(T_H - T_L)(s_2 - s_1)} = \frac{T_L}{T_H - T_L}
\]

\[
COP_{HP,\text{Carnot}} = \frac{T_H (s_2 - s_1)}{(T_H - T_L)(s_2 - s_1)} = \frac{T_H}{T_H - T_L}
\]
The reversed Carnot cycle is not a suitable model for refrigeration cycle!

- Process 2 – 3 involves the compression of a liquid-vapor mixture, which requires a compressor that will handle two phase.
- Process 4 – 1 involves the expansion of high-moisture-content refrigerant in a turbine.
Ideal Vapor-Compression Refrigeration Cycle

Process Description

1-2 Isentropic compression
2-3 Constant pressure heat rejection in the condenser
3-4 Throttling in an expansion valve
4-1 Constant pressure heat addition in the evaporator
Condenser $Q_H$

Evaporator $Q_L$

Expansion valve

Compressor
Energy analysis

From 1\textsuperscript{st} and 2\textsuperscript{nd} Law analysis for steady flow

<table>
<thead>
<tr>
<th>Component</th>
<th>Process</th>
<th>First law results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor</td>
<td>$s = \text{const.}$</td>
<td>$\dot{W}_{in} = \dot{m}(h_2 - h_1)$</td>
</tr>
<tr>
<td>Condenser</td>
<td>$P = \text{const.}$</td>
<td>$\dot{Q}_H = \dot{m}(h_2 - h_3)$</td>
</tr>
<tr>
<td>Throttle valve</td>
<td>$\Delta s &gt; 0$</td>
<td>$h_4 = h_3$</td>
</tr>
<tr>
<td></td>
<td>$\dot{W}_{net} = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\dot{Q}_{net} = 0$</td>
<td></td>
</tr>
<tr>
<td>Evaporator</td>
<td>$P = \text{const.}$</td>
<td>$\dot{Q}_L = \dot{m}(h_1 - h_4)$</td>
</tr>
</tbody>
</table>

\[
COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{h_1 - h_4}{h_2 - h_1}
\]

\[
COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{net,in}} = \frac{h_2 - h_3}{h_2 - h_1}
\]
Refrigerant-134a is the working fluid in an ideal compression refrigeration cycle. The refrigerant leaves the evaporator at -20°C and has a condenser pressure of 0.9 MPa. The mass flow rate is 3 kg/min. Find COP\textsubscript{R} and COP\textsubscript{R, Carnot} for the same \( T_{\text{max}} \) and \( T_{\text{min}} \), and the tons of refrigeration.

Use the Refrigerant-134a Tables

\[
\begin{align*}
\text{State 1} \\
\text{Compressor inlet} \\
T_1 &= -20^\circ C \\
x_1 &= 1.0 \\
h_1 &= 238.41 \ \frac{kJ}{kg} \\
s_1 &= 0.9456 \ \frac{kJ}{kg \cdot K}
\end{align*}
\]
State 2
Compressor exit
\[ P_{2s} = P_2 = 900 \text{ kPa} \]
\[ s_{2s} = s_1 = 0.9456 \frac{kJ}{kg \cdot K} \]
\[ h_{2s} = 278.23 \frac{kJ}{kg} \]
\[ T_{2s} = 43.79^\circ C \]

State 3
Condenser exit
\[ P_3 = 900 \text{ kPa} \]
\[ x_3 = 0.0 \]
\[ s_3 = 0.3738 \frac{kJ}{kg \cdot K} \]
\[ h_3 = 101.61 \frac{kJ}{kg} \]

State 4
Throttle exit
\[ T_4 = T_1 = -20^\circ C \]
\[ h_4 = h_3 \]
\[ x_4 = 0.358 \]
\[ s_4 = 0.4053 \frac{kJ}{kg \cdot K} \]
The tons of refrigeration (often called the cooling load or refrigeration effect)

\[ \dot{Q}_L = m(h_1 - h_4) \]

\[ = 3 \frac{kg}{min} \cdot (238.41 - 101.61) \frac{kJ}{kg} \cdot \frac{1 Ton}{211 \frac{kJ}{min}} \]

\[ = 1.94 Ton \]
Another measure of the effectiveness of the refrigeration cycle is how much input power to the compressor, in horsepower, is required for each ton of cooling.

The unit conversion is 4.715 hp per ton of cooling.

\[
\frac{\dot{W}_{\text{net, in}}}{\dot{Q}_L} = \frac{4.715}{COP_R} = \frac{4.715 \text{ hp}}{3.44 \text{ Ton}} = 1.37 \frac{\text{hp}}{\text{Ton}}
\]
Actual Vapor-Compression Refrigeration Cycle

Irreversibilities in various components
- Pressure drop due to fluid friction
- Heat transfer from or to surroundings
Refrigerant-134a enters the compressor of a refrigerator as superheated vapor at 0.14 MPa and -10°C at a rate of 0.05 kg/s and leaves at 0.8 MPa and 50°C. The refrigerant is cooled in the condenser to 26°C and 0.72 MPa and is throttled to 0.15 MPa. Disregarding any heat transfer and pressure drops in the connecting lines between the components, determine

(a) the rate of heat removal from the refrigerated space and the power input to the compressor,
(b) the isentropic efficiency of the compressor, and
(c) the coefficient of performance of the refrigerator.
Assumptions

1. Steady operating conditions
2. ΔKE & ΔPE are negligible

From: R-134a Table

\[
P_1 = 0.14 \text{ MPa} \quad \begin{align*} h_1 &= 246.36 \frac{\text{kJ}}{\text{kg}} \\ T_1 &= -10 \degree \text{C} \end{align*}
\]

\[
P_2 = 0.8 \text{ MPa} \quad \begin{align*} h_2 &= 286.69 \frac{\text{kJ}}{\text{kg}} \\ T_2 &= 50 \degree \text{C} \end{align*}
\]

\[
P_3 = 0.72 \text{ MPa} \quad \begin{align*} h_3 &= h_f @ 26 \degree \text{C} = 87.83 \frac{\text{kJ}}{\text{kg}} \\ T_3 &= 26 \degree \text{C} \end{align*}
\]

\[h_4 = h_3 \quad \text{(Throttling)} \rightarrow h_4 = 87.83 \frac{\text{kJ}}{\text{kg}}\]
(a) Rate of heat removal from the refrigerated space
\[ \dot{Q}_c = m(c_{\text{p},1} - h_4) \]
\[ = (0.05 \text{ kg/s}) \left( 246.36 - 87.83 \right) \text{kJ/kg} = 7.93 \text{ kW} \]

The power input to the compressor
\[ \dot{W}_{\text{in}} = m(c_{\text{p},2} - h_1) \]
\[ = (0.05 \text{ kg/s}) \left( 286.9 - 246.36 \right) \text{kJ/kg} = 2.02 \text{ kW} \]

(b) The isentropic efficiency of the compressor
\[ \eta_c = \frac{h_{25} - h_1}{h_{2} - h_1} \]

where at \( P_{25} = 0.8 \text{ MPa} \) and \( s_{25} = s_1 = 0.9924 \text{ kJ/kg.K} \)
\[ h_{25} = 284.21 \text{ kJ/kg} \]
Thus $\eta_c = \frac{284.21 - 246.36}{286.69 - 246.36} = 0.939$ or 93.9%.

(c) The coefficient of performance of refrigerator

$$\text{COP}_R = \frac{Q_c}{W_{in}}$$

$$= \frac{7.93 \text{ kW}}{2.02 \text{ kW}} \approx 3.93$$
Cascade refrigeration systems

Due to very low temperature, the temperature range in a single vapor compression refrigeration cycle become very large, Resulting low COP

$$COP_R = \frac{\text{Cooling effect}}{\text{Work input}} = \frac{Q_L}{W_{\text{net,in}}}$$

- Increase COP$_R$ by decreasing work input or increasing heat remove
Cascade refrigeration systems

Very low temperatures can be achieved by operating two or more vapor-compression systems in series, called cascading.

\[ \dot{m}_A (h_5 - h_8) = \dot{m}_B (h_2 - h_3) \]

\[ \dot{m}_A = \frac{(h_2 - h_3)}{(h_5 - h_8)} \dot{m}_B \]

\[ COP_{R,cascade} = \frac{\dot{Q}_L}{W_{net,in}} = \frac{\dot{m}_B (h_1 - h_4)}{\dot{m}_A (h_6 - h_5) + \dot{m}_B (h_2 - h_1)} \]
Air conditioning system

- Evaporator
- Expansion valve
- Condenser
- Pump
- Cooling tower
Example

Consider a two-stage cascade refrigeration system operating between the pressure limits of 0.8 and 0.14 MPa. Each stage operates on an ideal vapor compression refrigeration cycle with refrigerant-134a as the working fluid. Heat rejection from the lower cycle to the upper cycle takes place in an adiabatic counterflow heat exchanger where both streams enter at about 0.32 MPa. (In practice, the working fluid of the lower cycle is at a higher pressure and temperature in the heat exchanger for effective heat transfer). If the mass flow rate of the refrigeration through the upper cycle is 0.05 kg/s, determine

(a) the mass flow rate of the refrigeration through the lower cycle,
(b) the rate of heat removal from the refrigerated space and the power input to the compressor, and
(c) the coefficient of performance of this cascade refrigerator.
From Table of R-134a

Assumptions

1. Steady operating conditions
2. ΔKE & ΔPE are negligible
3. Heat exchanger is adiabatic

(a) Mass flow rate of the refrigerant through the lower cycle

Consider at the adiabatic heat exchanger

\[ E_{\text{out}} = E_{\text{in}} \rightarrow m_Ah_5 + m_Bh_3 = m_Ah_8 + m_Bh_2 \]

\[ m_A(h_5 - h_8) = m_B(h_2 - h_3) \]

\[ (0.05 \frac{kg}{\text{s}})(251.88 - 95.47) \frac{kJ}{kg} = m_B(255.93 - 55.16) \frac{kJ}{kg} \]
\[ m_B = 0.039 \text{ kg/s} \]

(b) The rate of heat removal by a cascade cycle is

the rate of heat absorption in the evaporator of
the lowest stage.

\[ Q_L = m_B (h_1 - h_4) \]

\[ = (0.039 \text{ kg/s})(239.16 - 55.16) \frac{\text{kJ}}{\text{kg}} = 7.18 \text{ kW} \]

The power input to a cascade cycle is the sum of
power inputs to all of the compressors.

\[ W_{in} = W_{comp1,in} + W_{comp2,in} \]

\[ = m_A (h_6 - h_5) + m_B (h_2 - h_1) \]

\[ = (0.05 \text{ kg/s})(270.92 - 251.89) \frac{\text{kJ}}{\text{kg}} + (0.039 \text{ kg/s})(255.93 - 239.16) \frac{\text{kJ}}{\text{kg}} \]

\[ = 1.61 \text{ kW} \]
(a) \[ \text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net,m}}} = \frac{7.18 \text{ kW}}{1.61 \text{ kW}} = 4.46 \]
When the fluid used throughout the cascade refrigeration system is the same, the heat exchanger between the stages can be replace by a mixing chamber, called a flash chamber.
Consider a two-stage compression refrigeration system operating between the pressure limits of 0.8 and 0.14 MPa. The working fluid is refrigerant-134a. The refrigerant leaves the condenser as a saturated liquid and is throttled to a flash chamber operating at 0.32 MPa. Part of the refrigerant evaporates during this flashing process, and this vapor is mixed with the refrigerant leaving the low pressure compressor. The mixture is then compressed to the condenser pressure by the high pressure compressor.
The liquid in the flash chamber is throttled to the evaporator pressure and cools the refrigerated space as it vaporizes in the evaporator. Assuming the refrigerant leaves the evaporator as a saturated vapor and both compressors are isentropic, determine

(a) the fraction of the refrigerant that evaporates as it is throttled to the flash chamber,

(b) the amount of heat removed from the refrigerated space and the compressor work per unit mass of refrigerant flowing through the condenser, and

(c) the coefficient of performance.
Assumptions
1. Steady operating conditions
2. ΔKE & ΔPE are negligible
3. Flash chamber is adiabatic

(a) The fraction of the refrigerant that evaporates as it is throttled to flash chamber is simply the quality at state 6

\[ x_6 = \frac{h_6 - h_f}{h_{fg}} = \frac{95.47 - 55.16}{196.71} = 0.2049 \]
(b) The amount of heat removed from the refrigerated space per unit mass of refrigerant flowing through the condenser is

\[ q_c = (1 - x_b)(h_1 - h_8) \]

The compressor work per unit mass

\[ \dot{w}_{in} = \dot{w}_{comp1,m} + \dot{w}_{comp2,m} \]

\[ = (1 - x_b)(h_2 - h_1) + (1)(h_4 - h_9) \]

From energy balance on the mixing chamber

\[ \dot{E}_{out} = \dot{E}_{in} \]

\[ (1) h_q = x_b h_3 + (1 - x_b) h_2 \]

\[ h_q = (0.2049)(251.88) + (1 - 0.2049)(255.93) \]

\[ = 255.10 \text{ kJ/kg} \]

\[ \text{and} \quad s_q = 0.9416 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad \text{(from table)} \]
At state 4; 0.8 MPa and $s_4 = s_g$, $h_4 = 274.48 \text{ kJ/kg}$

$$w_{in} = (1 - 0.2049)(268.93 - 239.16) \text{ kJ/kg}$$

$$+ (274.48 - 255.10) \text{ kJ/kg}$$

$$= 32.91 \text{ kJ/kg}$$

(5) \quad COP = \frac{q_c}{w_{in}} = \frac{146.3 \text{ kJ/kg}}{32.7 \text{ kJ/kg}} = 4.47