Lab # 2 System Modeling by MATLAB

What is a system ...?

In general, the system is a collection of some elements and components that be organized to take an input and give an output according to it. For example of a system is the DC motor that consist of coils and magnets and it take voltage as input and give a specific motion as output, so it is a system that transforms the electric power to a mechanical power.

What is a plant ...?

Typically, control engineers begin by developing a mathematical description of the dynamic system that they want to control. The system to be controlled is called a plant. As an example of a plant, this section uses the DC motor and develops the differential equations that describe the electromechanical properties of a DC motor with an inertial load and I want to design a controller to its speed so the motor is called a plant.

What is modeling ...?

When we go to the real practical work we only see physical devices and our job here is to deal with these physical devices and we know that all our dealing in university was with mathematical equations, so here we want a translator of the physical devices to a mathematical presentation. Modeling is the translator that represents a physical device by its equivalent equations.

In modeling we need some standard physical laws that relate our variables. For example we know that relation between the voltage and current across a resistor is proportional directly and related by Ohm's law $V=KI$. K : is constant (resistor).

Model Representation:

The plant of a system can be represented by:

1. Transfer Function $H(s)$, for example

\[ H(s) = \frac{s + 2}{s^2 + s + 10} \]

2. State Space Model (SS): is a mathematical model that consist of simultaneous, first order differential equations and an output equation, for example

\[ \frac{dx}{dt} = Ax + Bu \]
\[ y = Cx + Du \]

Where A, B, C, and D are matrices of appropriate dimensions, x is the state vector, and u and y are the input and output vectors.
Step and Impulse Responses:

- **Step Response**: Is the output of the system when the input is a unit step input.
- **Impulse Response**: Is the output of the system when the input is a unit impulse input.

Example:

Find the step response and impulse response for the following transfer functions by two methods

a) By Writing M-file

\[ G_1 = \frac{2S + 3}{S^2 + 3S - 2} \quad \text{and} \quad G_2 = \frac{4}{2S + 5} \]

b) Using Simulink

MATLAB Code

```matlab
clear all
clc
tf1 = tf([2 3],[1 3 -2])
tf2 = tf([4],[2 5])
figure(1)
subplot(2,1,1);step(tf1)
subplot(2,1,2);impulse(tf1)

Figure(2)
subplot(2,1,1);step(tf2)
subplot(2,1,2);impulse(tf2)
```

Simulink

![Simulink Diagram](image)

Results:
First Transfer Function:

![Graph of First Transfer Function]

Second Transfer Function:

![Graph of Second Transfer Function]

Polynomial Roots and Characteristic Polynomial

If \( p \) is a row vector containing the coefficients of a polynomial, \( \text{roots}(p) \) returns a column vector whose elements are the roots of the polynomial. If \( r \) is a column vector containing the roots of a polynomial, \( \text{poly}(r) \) returns an arrow vector whose elements are the coefficients of the polynomial.

To find the roots of following polynomial

\[ S^6 + 9S^5 + 31.25S^4 + 61.25S^3 + 67.75S^2 + 14.75S + 15 \]
The polynomial coefficients are entered in a row vector in descending powers
The roots are found using \texttt{roots}.

\textbf{MATLAB commands}

\begin{verbatim}
p=[ 1 9 31.25 61.25 67.75 14.75 15]
r=roots(p)
\end{verbatim}

The polynomial roots are obtained in column vector

\begin{verbatim}
r =
-4.0000
-3.0000
-1.0000 + 2.0000i
-1.0000 - 2.0000i
0.0000 + 0.5000i
0.0000 - 0.5000i
\end{verbatim}

If we have roots and we want to find the coefficients For example

\textbf{MATLAB commands}

\begin{verbatim}
r=[-1 -2 -3+4*i -3-4*i]
p=poly(r)
\end{verbatim}

\textbf{Results:}

\begin{verbatim}
r =
-1.0000  -2.0000  -3.0000 + 4.0000i  -3.0000 - 4.0000i

p =
     1   9  45  87  50
\end{verbatim}

Therefore, the polynomial equation is

\[ S^4 + 9S^3 + 45S^2 + 87S + 50 \]
Polynomial Fitting curves
In general, a polynomial fit to data in vector $x$ and $y$ is a function $p$ of the form

$$p(x) = c_1x^d + c_2x^{d-1} + ... + c_n$$

The degree is $d$, and number of coefficients is $n=d+1$. Given a set of points in vectors $x$ and $y$, polyfit($x,y,d$) returns the coefficients of $d$th order polynomial in descending powers of $x$.

As an example assume that you have below data

$x = \{0,1,2,4,6,10\}$, $y = \{1,7,23,109,307,1231\}$

We can solve it by write this on the MATLAB

```matlab
x=[0 1 2 4 6 10]
y=[1 7 23 109 307 1231]
c1=polyfit(x,y,1)
c2=polyfit(x,y,2)
c3=polyfit(x,y,3)
```

The coefficients of the degrees are found as follows

```
c1 =
119.2542  -177.4746

c2 =
 16.8768  -49.2275   26.7630

c3 =
     1.0000   2.0000   3.0000   1.0000
```

From coefficients we find that the most suitable polynomial is the $3^{rd}$ degree

Polynomial Evaluation
If $c$ is a vector whose elements are the coefficients of a polynomial in descending powers, the polyval ($c,x$) is the value of the polynomial evaluated at $x$. For example, to evaluate the above polynomial at points $0,1,2,3$ and $4$, use the commands

```
c=[1 2 3 1];
w=0:1:4;
z=polyval(c,w)
```
Exercises

Exercise (1): for the following two transfer functions:

\[ B(s) = \frac{s^2 + 5}{s^3 + 3s^2 + 5s + 8} \]
\[ G(s) = \frac{s + 5}{s^2 + 5s + 6} \]

Find the step response by two methods: a) Using m-file b) Using Simulink

Exercise (2): Find the polynomial whose roots are:

\[ R = [-1, -2, -3+4*i, -3-4*i, 3-6*i, 3+6*i, -3, -5] \]

Then evaluate the polynomial at the following points: 2, 3, 4, 5, 6, 7, 8.

Exercise (3): If you have the following values of x and y, find the suitable polynomial that describes this relation

- \( x = \{3, 5, 8, 12\} \), \( y = \{4, 7, 2, 6\} \)
- \( x = \{1, 3, 5, 7, 10, 25, 40\} \), \( y = \{-12, 3, 5, 9, 120, -4, 35\} \)
- \( x = \{0, 1, 2, 3, 4, 5\} \), \( y = \{0, 20, 60, 68, 77, 110\} \)

Exercise (4): You have the following block diagram

a) Find the state space of the system?

b) Find the transfer function of the system?