**Lab # 3 Time Response Design**

**State Space and Transfer Functions**

There are several different ways to describe a system of linear differential equations. The state-space representation is given by the equations:

\[
\frac{dx}{dt} = Ax + Bu
\]
\[
y = Cx + Du
\]

Where x is an n by 1 vector representing the state variable, u is a scalar representing the input, and y is a scalar representing the output. The matrices A (n by n), B (n by 1), and C (1 by n) determine the relationships between the state variable, input and output variables. Note that there are n first-order differential equations. State space representation can also be used for systems with multiple inputs and multiple outputs (MIMO), but we will only use single-input, single-output (SISO) systems in this experiment.

**MATLAB Commands:**

- **conv**: This command is used to convolve two polynomials. It is particularly useful for determining the expanded coefficients for factored polynomials. For example, this command can be used to enter the transfer function
  \[H(s) = \frac{s + 2}{(s + 1)(s - 3)}\]  
  by typing “H=tf([1 2],conv([1 1],[1 -3]))”.

- **series or ***: This command is used to combine two transfer functions that are in series. For example, if H(s) and G(s) are in series, they could be combined with the command “T=G*H” or “T=series(G,H)”.

![Series Connection Diagram](image)

```
>> num1=[1 2];den1=[1 2 3];num2=[1 3];den2=[1 -4 1];
>> [num,den]=series(num1,den1,num2,den2)
num =
   0   0   1   5   6
den =
   1  -2  -4  -10   3
>> G=tf(num,den)
Transfer function:
s^2 + 5 s + 6
-----------------------
s^4 - 2 s^3 - 4 s^2 - 10 s + 3
```
**parallel**: This command is used to combine two transfer functions that are in parallel. For example, if \( G(s) \) is in the forward path and \( H(s) \) is in the feedback path, they could be combined with the command “\( T=\text{parallel}(G,H) \)”. 

![Parallel System Diagram]

\[
\begin{align*}
\text{num} &= [1 2]; \text{den} = [1 2 3]; \text{num}2 = [1 3]; \text{den}2 = [1 -4 1]; \\
\text{G} &= \text{tf(rows,den)} \\
\text{Transfer function:} \\
\frac{s+2}{s^2+2s+3} + \frac{s+3}{s^2-4s+1} \\
\end{align*}
\]

**feedback**: This command is used to combine two transfer functions that are in feedback. For example, if \( G(s) \) is in the forward path and \( H(s) \) is in the feedback path, they could be combined with the command “\( T=\text{feedback}(G,H) \)”. 

![Feedback System Diagram]

\[
\begin{align*}
\text{num} &= [1 2]; \text{den} = [1 2 3]; \text{num}2 = [1 3]; \text{den}2 = [1 -4 1]; \\
\text{G} &= \text{tf(rows,den)} \\
\text{Transfer function:} \\
\frac{s+2}{s^2+2s+3} - \frac{s+3}{s^2-4s+1} \\
\end{align*}
\]
ss: this command is used to represent a system by state space model and it takes the four parameter matrices A,B,C,D. For example:

```matlab
sys = ss(A,B,C,D)
```
it converts the matrices and defines them as a system.

tf2ss: converts the parameters of a transfer function representation of a given system to those of an equivalent state-space representation. For example:

```matlab
[A, B, C, D] = tf2ss(num, den)
```
returns the A, B, C, and D matrices of a state space representation for the single-input transfer function.

ss2tf: converts a state-space representation of a given system to an equivalent transfer function representation. For example:

```matlab
[num,den] = ss2tf(A,B,C,D)
```
returns the transfer function.

linmod: obtains linear models from systems described as simulink models. Inputs and outputs are denoted in simulink block diagrams using inport and outport blocks. For example:

```matlab
[A,B,C,D]=linmod('controll_Lab3_1')
```
returns the state space representation of the simulink model which is previously saved as "controll_Lab3_1".

Example: for the following block diagram

- a) Find the state space and the transfer function of the system mathematically?
- b) Draw the step response of the system using simulink?
- c) Use the command "linmod" to export the data from simulink to m-file and then find the step response from the exported data?
- d) Compare between the results using simulink and the results using m-file?
Mathematically:

\[
X_1(t) = X_2(t) \ldots (1)
\]

\[M(s) = sX_2(s) + 5X_2(s) \xrightarrow{\text{yields}} m(t) = X_1(t) + 5X_2(t) \ldots (\star)\]

\[m(t) = 3X_2(t) + 6X_1(t) + U(t) \text{ then substitute in } (\star) \]

\[\xrightarrow{\text{yields}} X_2(t) = 6X_1(t) - 2X_2(t) + U(t) \ldots (2)\]

\[y(t) = -3X_1(t) + 8X_2(t) + 4U(t) \ldots (3)\]

Substitute in (1), (2) and (3) \(\xrightarrow{\text{yields}}\)

\[
\begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U(t)
\]

\[y(t) = \begin{bmatrix} -3 & 8 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + [4] U(t)\]

\[A = \begin{bmatrix} 0 & 1 \\ 6 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -3 & 8 \end{bmatrix}, d = 4\]

Transfer Function = \(C(SI - A)^{-1}B + d = \frac{4S^2 + 16S - 27}{S^2 + 2S - 6}\)

Using Simulink:

![Simulink Diagram](image-url)
**Using M-file:**
First change the step input type to import input type and then save the simulink. After that write the following code in new m-file and run it to get the following results.

**MATLAB Code**

```matlab
clear all
cle
[A,B,C,D]=linmod('controll_Lab3_1');
[num,den] = ss2tf(A,B,C,D);
X = tf(num,den)
step(X)
```

**Results:**

Note that the step response is the same in both simulink and m-file.

**Exercise:** for the following block diagram:

![Block Diagram](image)

a) Mathematically, find the state space of the system using the state variables X1 and X2?
b) From the state space found in part (a); find the transfer function of the system mathematically?
c) For the second block diagram; find the open loop transfer function G(s) which will make the two block diagrams equal?
d) Now connect the two block diagrams using simulink and using the transfer function found in part (c); after that check your result by finding the step response for the two block diagrams (note that the step response must be the same for the two block diagrams)?
e) For the first block export the results to workspace and use it to find the transfer function and draw the step response of the first block?
f) Do the same for the second block diagram and then check your results?