Basic Statistics ECON 1302

Practice Problems for Chapter 4

Section I: Multiple-Choice

1. A basketball player makes 160 out of 200 free throws. We would estimate the probability that the player makes his next free throw to be
   A) 0.16.
   B) 50−50. Either he makes it or he doesn't.
   C) 0.80.
   D) 1.2.

2. Suppose we roll a red die and a green die. Let A be the event that the number of spots showing on the red die is three or less and B be the event that the number of spots showing on the green die is more than three. The events A and B are
   A) disjoint.
   B) complements.
   C) independent.
   D) reciprocals.

3. In a particular game, a fair die is tossed. If the number of spots showing is either 4 or 5 you win $1; if the number of spots showing is 6 you win $4; and if the number of spots showing is 1, 2, or 3 you win nothing. If it costs you $1 to play the game, the probability that you win more than the cost of playing is
   A) 0.
   B) 1/6.
   C) 1/3.
   D) 2/3.

4. Event $A$ occurs with probability 0.3. If event $A$ and $B$ are disjoint then
   A) $P(B) \leq 0.3$.
   B) $P(B) \geq 0.3$.
   C) $P(B) \leq 0.7$.
   D) $P(B) \geq 0.7$.

5. A probability model must satisfy which of the following?
   A) The probability of any event must be a number between 0 and 1, inclusive.
   B) The sum of all the probabilities of all outcomes in the sample space must be exactly 1.
   C) The probability of an event is the sum of the outcomes in the sample space that make up the event.
   D) All of the above.
Does listening to soothing music help employees concentrate better? A psychologist studied the number of puzzles employees of a large business were able to solve in a five-minute period while listening to soothing music. Let $X$ be the number of puzzles completed successfully by a subject. The psychologist found that $X$ had the following probability distribution.

<table>
<thead>
<tr>
<th>Value of $X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

6. The probability that a randomly chosen employee completes at least three puzzles in the five-minute period while listening to soothing music is
   A) 0.3.
   B) 0.4.
   C) 0.6.
   D) 0.9.

7. $P(X < 3)$ has value
   A) 0.3.
   B) 0.4.
   C) 0.6.
   D) 0.9.

The probability density of a random variable $X$ is given in the figure below.

8. From this density, the probability that $X$ is between 0.5 and 1.5 is
   A) 1/3.
   B) 1/2.
   C) 3/4.
   D) 1.

9. The probability that $X$ is at least 1.5 is
   A) 0.
   B) 1/4.
   C) 1/3.
   D) 1/2.
10. The probability that $X = 1.5$ is
   A) 0.
   B) 1/4.
   C) 1/3.
   D) 1/2.

11. In a particular game, a fair die is tossed. If the number of spots showing is either 4 or 5 you win $1; if the number of spots showing is 6 you win $4; and if the number of spots showing is 1, 2, or 3 you win nothing. Let $X$ be the amount that you win. The expected value of $X$ is
   A) $0.00$.
   B) $1.00$.
   C) $2.50$.
   D) $4.00$.

12. Suppose $X$ is a random variable with mean $\mu_X$ and standard deviation $\sigma_X$. Suppose $Y$ is a random variable with mean $\mu_Y$ and standard deviation $\sigma_Y$. The mean of $X + Y$ is
   A) $\mu_X + \mu_Y$.
   B) $(\mu_X/\sigma_X) + (\mu_Y/\sigma_Y)$.
   C) $\mu_X + \mu_Y$, but only if $X$ and $Y$ are independent.
   D) $(\mu_X/\sigma_X) + (\mu_Y/\sigma_Y)$, but only if $X$ and $Y$ are independent.

13. Suppose $X$ is a random variable, with mean $\mu_X$ and standard deviation $\sigma_X$. Suppose $Y$ is a random variable, with mean $\mu_Y$ and standard deviation $\sigma_Y$. The variance of $X + Y$ is
   A) $\sigma_X + \sigma_Y$.
   B) $(\sigma_X)^2 + (\sigma_Y)^2$.
   C) $\sigma_X + \sigma_Y$, but only if $X$ and $Y$ are independent.
   D) $(\sigma_X)^2 + (\sigma_Y)^2$, but only if $X$ and $Y$ are independent.

A small store keeps track of the number $X$ of customers that make a purchase during the first hour that the store is open each day. Based on the records, $X$ has the following probability distribution.

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X)$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.6</td>
</tr>
</tbody>
</table>

14. The mean number of customers that make a purchase during the first hour that the store is open is
   A) 2.0.
   B) 2.5.
   C) 3.0.
   D) 4.0.
15. The standard deviation of the number of customers that make a purchase during the first hour that the store is open is
   A) 1.4.
   B) 2.0.
   C) 3.0.
   D) 4.0.

16. Suppose the store is open seven days per week from 8:00 AM−5:30 PM. The mean number of customers that make a purchase during the first hour that the store is open during a one-week period is
   A) 3.0.
   B) 9.0.
   C) 21.0.
   D) 28.0.

17. A random sample of size 25 is to be taken from a population that is normally distributed with mean 60 and standard deviation 10. The average \( \bar{x} \) of the observations in our sample is to be computed. The sampling distribution of \( \bar{x} \) is
   A) normal with mean 60 and standard deviation 10.
   B) normal with mean 60 and standard deviation 2.
   C) normal with mean 60 and standard deviation 0.4.
   D) normal with mean 12 and standard deviation 2.

18. The scores of individual students on the American College Testing (ACT) Program composite college entrance examination have a normal distribution with mean 18.6 and standard deviation 6.0. At Northside High, 36 seniors take the test. If the scores at this school have the same distribution as national scores, what is the mean of the sampling distribution of the average (sample mean) score for the 36 students?
   A) 1.0
   B) 3.1
   C) 6.0
   D) 18.6

19. The scores of individual students on the American College Testing (ACT) Program composite college entrance examination have a normal distribution with mean 18.6 and standard deviation 6.0. At Northside High, 36 seniors take the test. If the scores at this school have the same distribution as national scores, what is the standard deviation of the sampling distribution of the average (sample mean) score for the 36 students?
   A) 1.0
   B) 3.1
   C) 6.0
   D) 18.6
20. Suppose that you are a student worker in the Accounting Department of the Business School and they agree to pay you using the Random Pay system. Each week the Chair flips a coin. If it comes up heads, your pay for the week is $80 and if it comes up tails your pay for the week is $40. Your friend is working for the engineering department and makes $65 per week. The probability that your total earnings in 100 weeks are more than hers is approximately
A) 0.0000.
B) 0.4013.
C) 0.5000.
D) 0.5987.

21. I take an SRS of size $n$ from a population that has mean 80 and standard deviation 20. How big should $n$ be so that the sampling distribution of $x$ has standard deviation 1?
A) 400
B) 20
C) Approximately 5
D) Cannot be determined unless we know the population follows a normal distribution.

22. The SAT scores of entering freshman at a certain university have a mean of 1200 and a standard deviation of 90. A random sample of 100 freshman is taken and $x$, the sample mean of their SAT scores, is computed. The sampling distribution of $x$ is
A) approximately $\mathcal{N}(1200, 90)$.
B) approximately $\mathcal{N}(1200, 9)$.
C) approximately $\mathcal{N}(1200, 0.9)$.
D) exactly $\mathcal{N}(1200, 0.9)$.

23. The SAT scores of entering freshman at a certain university have mean 1215 and standard deviation 110. A random sample of 100 freshman is taken and $x$, the sample mean of their SAT scores, is computed. The probability that $x$ is less than 1190 is
A) 0.0116.
B) 0.1335.
C) 0.4090.
D) 0.4562.

24. The number of phone calls your business receives daily has mean 20.6 and standard deviation 4.4. You record the number of phone calls received over the next 100 days and compute the average $x$. The probability that $x$ is more than 21 is
A) approximately 0.9186.
B) approximately 0.1814.
C) approximately 0.
D) unknown. It cannot be computed because we do not know the distribution of the number of phone calls per day.
Section II: Free Response Problems

1. The life expectancy in the United States is 75 with a standard deviation of 7 years. A random sample of 49 individuals is selected.
   a. What is the probability that the sample mean will be larger than 77 years?
      0.0228
   b. What is the probability that the sample mean will be less than 72.7 years?
      0.0107
   c. What is the probability that the sample mean will be between 73.5 and 76 years?
      0.7745
   d. What is the probability that the sample mean will be between 72 and 74 years?
      0.1573
   e. What is the probability that the sample mean will be larger than 73.46 years?
      0.9382

2. You have two scales for measuring weights in a chemistry lab. Both scales give answers that vary a bit in repeated weightings of the same item. If the true weight of a compound is 2 grams (g), the first scale produces readings X that have mean 2.0 g and standard deviation .002 g. The second scale produces readings Y that have mean 2.01 g and standard deviation .01 g.
   a. What are the mean and the standard deviation of the difference 2Y-3X? (The readings X and Y are independent).
   (b) You measure once with each scale and average the readings. Your result is Z = (X+Y)/2. What are \( \mu_z \) and \( \sigma_z \)?