1. Introduction

When the ratio \((L/S)\) is less than 2.0, slab is called two-way slab, as shown in the fig. below. Bending will take place in the two directions in a dish-like form. Accordingly, main reinforcement is required in the two directions.
2. Types Of Two Way Slabs

- Slabs without beams
  - Flat plates
  - Flat slabs
- Slabs with beams
  - Two-way edge-supported slab
  - Two-way ribbed slab
  - Waffle slabs
  - Two-way Edge-supported ribbed slabs

3. Design Methods

- Simplified Design Methods
  - Grashoff method
  - Marcus method
  - Egyptian Code method
- Direct Design Method "DDM"
- Equivalent Frame Method " EFM "

Design methods
4. Direct Design Method "D.D.M"

Before discussion of this Method, we have to study some concepts:

1. **Limitations:**

   1. Three or more spans in each direction.
   2. Variation in successive spans $\geq 33\% \left( \frac{l_2-l_1}{l_2} \times 100\% \geq 33\% \right)$.
   3. $L - L \neq 2 \times D L$
   4. Column offset $\neq 10\%$ in each direction.
   5. $L/B \neq 2$.
   6. For slabs on beams, for one panel $\frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} \ll 0.2$
      
      $\gg 5.0$.

2. **Determination of Two way slab thickness:**

   - **Case 1**: interior and edge beams are exist.

   \[ h_{\text{min}} = \frac{l_n(0.8 + f_y/14000)}{36 + 5\beta(1 + \beta_s)} \]
   \[ h_{\text{max}} = \frac{l_n(0.8 + f_y/14000)}{36} \]

   Where:

   - $l_n$: is the largest clear distance in the longest direction of panels.
   - $S_n$: is the clear distance in the short direction in the panel.
   - $B = \frac{L_n}{S_n}$
   - $B_s = \frac{\text{length of continues edges in the panel}}{\text{Total perimeter of the panel}}$

   **Example for finding $B_s$**: for fig. shown:

   For panel 1 $B_s = \frac{l_1 + s}{2l + 2s} = \frac{1}{2}$
   For panel 5 $B_s = \frac{2l_1 + 2s}{2l + 2s} = 1$

   So $h$ to be used should be: $h_{\text{min}} < h < h_{\text{max}}$
• **Case 2**: interior beams are not existing, thickness can be found according to table 8.8, page 339.

**Table 8.8: Minimum thickness of slabs without interior beams**

<table>
<thead>
<tr>
<th>Without drop panels</th>
<th>With drop panels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior panels</td>
<td>Interior panels</td>
</tr>
<tr>
<td>Without edge beams</td>
<td>With edge beams</td>
</tr>
<tr>
<td>( \ell_n / 30 )</td>
<td>( \ell_n / 33 )</td>
</tr>
</tbody>
</table>

\(^* f_y = 4200 \text{ kg/cm}^2\)

3. **Estimating dimensions of interior and exterior beams sections:**

Dimensions can be estimated from the following figures:

Where:  
\( b = \) beam width,  
\( h = \) slab thickness,  
\( a = \) beam thickness.

\[ b + 2(a - h) \leq b + 8h \]
\[ a - h \leq 4h \]

\[ b + (a - h) \leq b + 4h \]
\[ a - h \leq 4h \]

*Figure 8.20: Effective beam section; (a) interior beam; (b) exterior beam*
Design Procedures

Discussion will be done to one representative strip in the horizontal and vertical directions; the same procedure can be used for the other strips.

a- Determination of total factored Static Moment $M_o$:

$$M_o = W_u \times \text{Strip width} \times \frac{l_n^2}{8}$$

where:

$W_u$ : total factored load in t/m$^2$.

$l_n$ = clear distance in the direction of strip, and not less than $0.65l_1$. 
b- Distribution of the total factored static moment to negative and positive moments:

I. For interior Spans:

According to the code, the moments can be distributed according to factors shown in the figure:

\[
\begin{align*}
0.65 M_o & \\
0.35 M_o & \\
0.65 M_o &
\end{align*}
\]

II. For Edge Spans:

**Table 8.9: Distribution of total static moment in end spans**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior negative</td>
<td>0.75</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>factored moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive factored</td>
<td>0.63</td>
<td>0.57</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exterior negative</td>
<td>0.00</td>
<td>0.16</td>
<td>0.26</td>
<td>0.30</td>
</tr>
<tr>
<td>factored moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Static Mom. $M_o$ can be distributed, according to factors given in the table 8.9, page 341.

\[
\begin{align*}
0.65 M_o & \\
0.35 M_o & \\
0.65 M_o &
\end{align*}
\]

(a) Exterior edge fully restrained

\[
\begin{align*}
0.16 M_o & \\
0.57 M_o & \\
0.70 M_o &
\end{align*}
\]

(c) beams between all supports

\[
\begin{align*}
0.30 M_o & \\
0.50 M_o & \\
0.70 M_o &
\end{align*}
\]

(e) No beams between interior supports and with edge beams

\[
\begin{align*}
0.26 M_o & \\
0.52 M_o & \\
0.70 M_o &
\end{align*}
\]

(d) No beams between interior supports and without edge beams

\[
\begin{align*}
0.63 M_o & \\
0.63 M_o & \\
0.75 M_o &
\end{align*}
\]

(b) Exterior edge unrestrained
c- Distribution of the positive and negative factored moments to the Column and middle strips:

![Diagram of Two-Way Slabs]

**Note:** width of column strip is equal to $0.25l_1$ or $0.25l_2$ which is **smaller**.

- $l_1$: length in the direction of strip, center to center between columns.
- $l_2$: length in the direction perpendicular to $l_1$.

I. Determination of factored moments on column and middle strips:

- Finding $\alpha$ and $\beta_t$:
  - $\alpha = \frac{I_b}{I_s}$
    - $\alpha$: is ratio of flexural stiffness.
    - $I_b$: Moment of inertia of the beam in the direction of strip... can be found from fig.8.14 and fig.8.15, pages 310 and 311.
    - $I_s$: Moment of inertia of slab $= \frac{1}{12} \times \text{strip width} \times h^3$, where $h$ is slab thickness.
  - $\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$
    - $\beta_t$: Ratio of torsional stiffness
    - $E_{cb}$ and $E_{cs}$ are the modulus of elasticity of concrete for beam and slab.

**Note:** $\alpha$ is given only for the beams in the direction of the strip.

**Note:** $\beta_t$ is given only for edge beams perpendicular to the strip.
C: Cross sectional constant defines torsional properties \[ C = \Sigma \left( 1 - 0.63 \frac{X}{Y} \right) \left( \frac{X^3Y}{3} \right) \]

X: smallest dimension in the section of edge beam.
Y: Largest dimension in the section of edge beam.

Note: the C relation is applicable directly for rectangular section only, but when used for L-Shape beams, we should divide it into two rectangular sections and find C.

\[ C" A" = C_1 + C_2 \text{ for } A \quad \text{and} \quad C" B" = C_1 + C_2 \text{ for } B. \]

C to be used = Max (C "A" , C "B" ).

When \( \alpha \) and \( \beta \) are found, factors for moment can be found from table 8.10 page 343 for the column strip.

<table>
<thead>
<tr>
<th>Table 8.10: Column strip factored moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_2/l_1 )</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Exterior negative factored moment</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Positive factored moment</td>
</tr>
<tr>
<td>Interior negative factored moment</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Notes:
- \( \alpha \ l_2/l_1 = 0.0 \), when there is no interior beams in the direction of strip under consideration.
- \( \beta = 0.0 \), when there is no exterior “edge” beams perpendicular to the strip under consideration.
After finding the moments on the column strip, Moments on the middle strip is the remain.

II. For the moment on the beam “if exist”:

If: \( \alpha \frac{l_2}{l_1} \geq 1 \) ... The beam moment is 85% of the moment of the column strip.

\( \alpha \frac{l_2}{l_1} = 0 \) ... there is no beam .. mom. = 0

\( 0 < \alpha \frac{l_2}{l_1} < 1 \) ... Interpolation have to be done between 0 and 85% to find percentage of moment on the beam from that of the column strip.

** The Mom. on the remain part of column strip = Tot. Mom. on the column strip – Mom. on the beam.

Summary:

1- Find \( M_0 \):

2- Distribute \( M_0 \) into +ve and –ve Mom.

3- Distribute Mom. Into column strip and Middle Strip.

4- Distribute Mom. In column strip into Mom. On beam and remained slab.

After calculating Moments, we can find the \( \rho \), then \( A_{st} \) required
Example 1:
For the given data, design strip 1-2-3-4 of the two way slab for flexure.

Data:

Columns are 30cm X 30cm, Equivalent partitions load=250 Kg/m², Live Load = 400Kg/m²,
$f'_c = 280 \text{ kg/cm}^2$, $f_y = 4200 \text{ Kg/cm}^2$, slab thickness = 16cm
Solution:

Thickness is given 16cm, no need to be checked.

1- Calculate total factored load $W_u \, \text{"t/m}^2\text{"}$:

$$W_u = 1.4(0.16 \times 2.5 + 0.25) + 1.7 \times 0.4 = 1.59 \, \text{t/m}^2.$$  

2- Determine The Total Factored Static Moment ($M_o$) :

$$M_o = \frac{W_u \times \text{strip width} \times l_n^2}{8} = \frac{1.59 \times 3.15 \times 4.7^2}{8} = 13.83 \, \text{t.m}$$
3- Distribute $M_o$ into $+ve$ and $-ve$ moments:

The total factored static moment was distributed according to Table "8.9" in your textbook as shown in the following Figure.
4- Moments on the column Strip :
Evaluate the constant $\alpha$ and $\beta$

- Evaluation of $\alpha$:
  \[
  \alpha = \left(\frac{b}{\text{strip width}}\right) \left(\frac{a}{h}\right)^3 f. \quad \text{"For beam in direction of strip"}
  \]
  For $a/h=50/16=3.125$ and $b/h=30/16=1.875, f=1.4$ (Fig. 8.14)
  \[
  \alpha = \left(\frac{0.3}{3.15}\right) (3.125)^3 \times 1.4 = 4.07
  \]

- Evaluation of $\beta$:
  \[
  \beta_t = \frac{C}{2l_s}. \quad \text{"For edge beam perpendicular to direction of strip"}
  \]
  \[
  C_A = \left(1 - 0.63 \times \frac{16}{24}\right) \left(\frac{16^3 \times 24}{3}\right) + \left(1 - 0.63 \times \frac{30}{40}\right) \left(\frac{30^3 \times 40}{3}\right) = 208,909.55 \text{cm}^4
  \]
  \[
  C_B = \left(1 - 0.63 \times \frac{16}{54}\right) \left(\frac{16^3 \times 54}{3}\right) + \left(1 - 0.63 \times \frac{24}{36}\right) \left(\frac{24^3 \times 30}{3}\right) = 128,532.48 \text{cm}^4
  \]
  \[
  C = \text{Max (} C_A \text{ or } C_B \text{)} = 208,909.55
  \]
  \[
  \beta_t = \frac{208,909.55}{12 \times 315 \times 16 \times 3 \times 2} = 0.97
  \]

After $\alpha$ and $\beta_t$ are calculated, factors for the moment of column strip can be found from Table 8.10, page 343

\[
\alpha \frac{l_2}{l_1} = 4.07 \times \frac{6}{5} = 4.88 > 1, \quad \beta_t \text{ between 0 and 2.5, } \frac{l_2}{l_1} = 1.2
\]

- Ve exterior moment Factor:

<table>
<thead>
<tr>
<th>$\frac{l_2}{l_1}$</th>
<th>$l_1/l_2=1$</th>
<th>$l_1/l_2=1.2$</th>
<th>$l_1/l_2=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_t = 0$</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$\beta_t = 0.97$</td>
<td>0.903</td>
<td>0.8797</td>
<td>0.7866</td>
</tr>
<tr>
<td>$\beta_t = 2.5$</td>
<td>75</td>
<td>69</td>
<td>45</td>
</tr>
</tbody>
</table>
• +Ve interior moment Factor:

<table>
<thead>
<tr>
<th>$l_1/l_2$</th>
<th>$l_1/l_2=1$</th>
<th>$l_1/l_2=1.2$</th>
<th>$l_1/l_2=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75</td>
<td>69</td>
<td>45</td>
</tr>
</tbody>
</table>

• -Ve interior moment Factor:

<table>
<thead>
<tr>
<th>$l_1/l_2$</th>
<th>$l_1/l_2=1$</th>
<th>$l_1/l_2=1.2$</th>
<th>$l_1/l_2=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75</td>
<td>69</td>
<td>45</td>
</tr>
</tbody>
</table>
5- Moments on the Middle Strip: "The remain moment":

Middle Strip Moments

6- Moment On Beam:

As $\alpha \frac{l_2}{l_1} > 1$ .... Beam will resist 85% of the column strip moment.

Beam Moments

7- Moment On Remained Slab:

Remain Slab Moments
Notes:
- For each value of moment, \( \rho \) can be calculated, then \( A_{st} \).
- **Widths to used for design and \( \rho \) calculations are:**
  - For the remained slab of column strip: \( b = 1.25 - 0.64 = 0.61 \) m
  - For half middle strip: \( b = 3.15 - 1.25 = 1.9 \) m
  - Beam is designed as rectangular section of \( b = 0.3 \) m at \( -ve \) moments and as T-section at \( +ve \) moments.
- Beam should be designed for shear, according to specifications of code **ACI 318"13.6.8"**, and reported in page 344 of your text book.

By:

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