Solve The following questions.

[Q.1 (5 Marks)]

(a) If $P(C_1) = 0.4$, $P(C_2) = 0.6$ and $P(C_1|C_2) = 0.5$. Find the following:

(i) $P(C_1 \cap C_2)$

(ii) $P(C_1^c \cap C_2^c)$

(iii) $P($Exactly one of the two events occurs$)$.

(b) Prove that for any two events $A$ and $B$,

$$P(A \cap B) \geq 1 - P(A^c) - P(B^c).$$
[Q.2 (5 Marks)] (a) Five cards are drawn at random from an ordinary deck. Find the probability that

(i) All the cards are from the same suit.

(ii) Three cards from a kind and the other two cards are from another kind.

(b) A box contains 4 red balls and 8 black balls. A sample of 3 balls is drawn at random from the box. Let $X$ denote the number of red balls in the sample.

(i) Find the pmf of $X$.

(ii) Find the probability that at least one red ball is in the sample.
(a) Suppose that the random variable $X$ has a cdf given by

$$F(x) = \begin{cases} 
0, & x < 0, \\
x^2, & 0 \leq x < 1, \\
1, & x \geq 1.
\end{cases}$$

Find $P(\frac{1}{2} < Y \leq \frac{3}{4})$ by two ways. First by using the cdf and second by using the pdf.

(b) Let $X$ be a discrete random variable with pmf

$$p_X(x) = \begin{cases} 
k\left(\frac{2}{3}\right)^x, & x = 2, 3, 4, \ldots \\
0, & \text{otherwise.}
\end{cases}$$

(a) Find the value of $k$ that makes $p_X(x)$ a pmf.

(b) If $Y = 2X + 1$. Find the pmf of $Y$.  

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(a) Prove that for any random variable $X$,

$$P(X = x) = F_X(x) - F_X(x^-), \quad \forall x \in \mathbb{R}.$$ 

(b) A manufacture buy 0.65 of the fuses from company I out of which 0.03 are defective and the remaining from company II out of which 0.02 are defective. If a fuse is chosen at random and it was defective, what is the probability that it was from company II?