Incremental Analysis

- Incremental Rate of Return Analysis
- Elements in Incremental Analysis
- Choosing an Analysis Method
Incremental Analysis

**Incremental analysis** was presented earlier in chapter 7 and was defined as the examination of the differences between alternatives. The incremental solution presented in chapter 7 was limited to two alternatives. There, we saw, if we had projects A and B with CFD’s:

A = {-20, 28}, B = {-10, 15},

we could define: C = A − B = {-10, 13}

And have: A = B + C ⇔

**Higher-cost alternative = lower-cost alternative + difference**

In this chapter, we will expand our analysis to include three or more alternatives. The approach is called **incremental analysis**, and builds on the analysis performed in Chapter 7.

Incremental analysis can be examined either **graphically** or **numerically**.
Incremental Analysis Using Graphical approach

Again, consider projects A and B with CFD’s:

\[ A = \{-20, 28\}, \quad B = \{-10, 15\}, \]

we could define: \( C = A - B = \{-10, 13\} \)

We first consider a graphical approach to incremental analysis.
For a given \( i=6\% \), calculate corresponding PWC and PWB for alternative and then we can plot Alt. B = (10, 14.15), Alt. A = (20, 26.42), as points in the PWC-PWB graph
Incremental Analysis Using Graphical approach

A = {-20, 28}, plotted as (20, 26.42)
B = {-10, 15}, plotted as (10, 14.15)
Incremental Analysis Using Graphical approach

► This graph is called the benefit-cost graph.
► At the chosen interest rate, the line at which $\text{PWB} = \text{PWC}$ (i.e. $\text{NPW} = 0$) divides the graph into areas of desirable and undesirable alternatives.
► Note that the line will be a 45° line if the vertical and horizontal scales are identical.
► An alternative plotted above the line is desirable since its PW of benefits exceeds its PW of costs (positive NPW).
► An alternative plotted below the line is undesirable since its PW of benefits is less than its PW of costs (negative NPW).
Incremental Analysis Using Graphical approach

A = {-20,28},  B = {-10,15}

Since Alt. A has CFD2 = {20,28}, its IRR is 40%.

Suppose we construct a line L from the origin through the point Alt. B = (20, 26.42).

Note L has slope m = 1.321.

We can associate the line L with all one-year projects having a CFD with an IRR of 40%.
Incremental Analysis

a) If \(-a, b\) is any CFD with an IRR of 40%,

\[
PWC = a, \quad \text{since the IRR} = 40\% \quad \Rightarrow \quad b = (1.4)(PWC) = (1.4)(a), \quad \text{and}
\]

\[
PWB = (1+i)^{-1}b = (1+0.06)^{-1}b = (0.9434)b = (0.9434)(1.4a) = 1.321a = 1.321PWC.
\]

Thus the slope of the line joining the origin and \((PWC,PWB)\) is \(PWB/PWC = 1.321\). Thus \((PWC,PWB)\) lies on the line \(L\).

b) Conversely, let \((PWC, PWB)\) lies on the line \(L\).

This means \(PWB/PWC = 1.321\), so \(PWB = 1.321PWC\). Let \(-a, b\) be the CFD resulting in \((PWC,PWB)\). This means

\[
PWC = a, \quad \text{PWB} = (0.9434)b = b/(1.06).
\]

Thus \((0.9434)b = 1.321a\), so \(b = 1.4a\).

This means the CFS \(-x,y\) for this alternative has an IRR of 40%.

Conclusion.

The projects with points \((PWC,PWB)\) lying on the line \(L\) are the ones whose CFD’s generate a 40% IRR. In particular, Alt. 2 has IRR = 40%.
Incremental Analysis

A = {-20,28},  B = {-10,15}

Recall Alt. B had IRR = 50%.
If we plot a line $L_{50\%}$ from the origin through the point for Alt. B, it would correspond to the projects whose CFS’s generate a 50% IRR.

Summary

- The projects with their (PWC,PWB) lying on the 50% line $L_{50\%}$ are the ones that have CFD’s with a 50% IRR.
- The projects with their (PWC,PWB) lying on the 40% line $L_{40\%}$ are the ones that have CFD’s with a 40% IRR.
- The projects with their (PWC,PWB) lying on the line NPW = 0 are the ones that have CFD’s with a 6% IRR.
Incremental Analysis

$A = \{-20, 28\}$,
$B = \{-10, 15\}$

On the plot:
$A = (20, 26.42)$,
$B = (10, 14.15)$

Difference = Alt. 2 – Alt. 1 = (10, 12.27).

We know that the slopes of lines correspond to the IRR’s of CFS’s of alternatives. S represents a “difference” alternative, and has a slope corresponding to an IRR of more than 6%, since its slope is greater than that of the 6% line, NPW = 0. Thus the difference alternative has an incremental rate of return of more than 6%. (Indeed, we could discover that S has the same slope as a 30% line.)

From earlier incremental analysis, we know that Alt. 2 is preferable to Alt. 1 if the incremental rate of return exceeds the MARR. In this case, 30% > 6%.

We can thus conclude that **Alt. 2 is preferable to Alt. 1 just by observing that the slope of the line segment S is greater than the slope of the 6% line.**
Example 8-3

Three alternatives ranked in order of increasing cost; all have a 20-year life, with no salvage value. MARR = 6%.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$2000</td>
<td>$4000</td>
<td>$5000</td>
</tr>
<tr>
<td>Uniform annual benefit</td>
<td>410</td>
<td>639</td>
<td>700</td>
</tr>
</tbody>
</table>

Which Project should you choose?

\[
\text{PW of benefits} = (\text{UAB})(P/A,6\%,20) = \text{UAB} (11.470)
\]

NPW analysis shows that Alt. B is the best (the last line in table 2)

Conclusion.

Each line length from an Alternative point to the 45-degree line is the NPW of the Alternative. The project with the greatest NPW is thus the one with the longest line dropped from it to the 45-degree line.
Incremental Analysis Using Numerical Approach

1. Be sure all the alternatives are identified. We must have all the mutually exclusive alternatives tabulated, including the do-nothing alternative.

2. (Optional) Compute the IRR for each alternative. If one or more alternatives has a ROR at least as large as the MARR, then we can discard those with ROR < MARR.

3. Arrange the remaining alternatives in ascending order of investment. Each difference we analyze should be a higher-cost alternative minus a lower-cost alternative.

4. Make a two-alternative analysis of the first two alternatives.
   \[(\text{Higher-cost Alt. Y}) = (\text{Lower-cost Alt. X}) + (Y-X)\]
   Compute \(\Delta ROR\) for \((Y-X)\), the increment of investment.
   If \(\Delta ROR \geq MARR\), choose Y. If not, choose X.
   Reject the alternative not chosen

5. Take the preferred alternative from step 4, and the next alternative from the list created in step 3. Proceed with another two-alternative comparison.

6. Continue until all alternatives have been examined and the best of the multiple alternatives has been identified.

Note: What if two alternatives have the same cost? This anomalous situation can occur in step 4.
   Choose the one so that the difference represents an increment of investment (check example 7-9)
In situations where an increment of borrowing is examined, the criterion is
   If \(\Delta ROR \leq MARR\), the increment is acceptable. If not, it is unacceptable.

Incremental Analysis With Unlimited Alternatives
   We will skip this section.
Example 8-4
Use the same data as in example 8-3 except that alternative A has a UAB of $122 instead of $410.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$2000</td>
<td>$4000</td>
<td>$5000</td>
</tr>
<tr>
<td>Uniform annual benefit</td>
<td><strong>122</strong></td>
<td>639</td>
<td>700</td>
</tr>
</tbody>
</table>

Which Project should you choose?
Example 8-6

Use the incremental rate of return to solve example 8-3 mathematically.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$2000</td>
<td>$4000</td>
<td>$5000</td>
</tr>
<tr>
<td>Uniform annual benefit</td>
<td><strong>410</strong></td>
<td>639</td>
<td>700</td>
</tr>
</tbody>
</table>
Example 8-7

Use the incremental rate of return to solve example 8-4 mathematically.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>$2000</td>
<td>$4000</td>
<td>$5000</td>
</tr>
<tr>
<td>Uniform annual benefit</td>
<td>122</td>
<td>639</td>
<td>700</td>
</tr>
</tbody>
</table>
Example 8-8
The following five alternatives have 20-year useful life. Which alternative should be selected using the incremental rate of return analysis?

a) Mathematically

b) Graphically (solution in Fig. 8-8 in your text)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$4000</td>
<td>$2000</td>
<td>$6000</td>
<td>$1000</td>
<td>$9000</td>
</tr>
<tr>
<td>UAB</td>
<td>$639</td>
<td>$410</td>
<td>$761</td>
<td>$117</td>
<td>$785</td>
</tr>
<tr>
<td>Rate of return</td>
<td>15%</td>
<td>20%</td>
<td>11%</td>
<td>10%</td>
<td>6%</td>
</tr>
<tr>
<td>PWB (calculated from line 2)</td>
<td>$7330</td>
<td>$4700</td>
<td>$8730</td>
<td>$1340</td>
<td>$9000</td>
</tr>
<tr>
<td>NPV (to check)</td>
<td>3330</td>
<td>2700</td>
<td>1730</td>
<td>340</td>
<td>0</td>
</tr>
</tbody>
</table>
Choosing an Analysis Method

We have now seen three major economic analysis techniques:
1. present worth analysis (PW analysis),
2. annual cash flow analysis (ACF analysis),
3. rate of return analysis (ROR analysis).

Which method should be used for any particular problem?
The following points are important:

✓ You must know the MARR to use PW analysis or ACF analysis.
✓ PW analysis and ACF are often simpler than ROR analysis.
✓ In some contexts, ROR is easiest to explain. In others, ACF analysis is easier to explain.
✓ The company you work for may dictate the analysis method you must use.
✓ ROR analysis is most often used in industry.
Chapter Summary (mathematical approach)

A benefit-cost graph (PW of benefits vs. PW of cost) can help with incremental analysis to choose between alternatives.

Important steps in incremental ROR analysis

1. Check to see all alternatives are identified.
2. (Optional) Compute the RR for each alternative. Alternatives with ROR < MARR can be immediately rejected.
3. Arrange remaining alternatives in ascending order of investment.
4. Make a two-alternative analysis for the first two alternatives.
5. Take the preferred alternative from Step 4, and the next alternative from the list in Step 3. Proceed with another two-alternative comparison.