**Horizontal Distribution of Forces to Individual Shear Walls**

**Interaction of Shear Walls with Each Other**

In the shown figure the slabs act as horizontal diaphragms extending between cantilever walls and they are expected to ensure that the positions of the walls, relative to each other, don't change during lateral displacement of the floors. The flexural resistance of rectangular walls with respect to their weak axes may be neglected in lateral load analysis.

The distribution of the total seismic load, $F_x$ or $F_y$ among all cantilever walls may be approximated by the following expressions.

\[
\begin{align*}
ixF' &= \text{load induced in wall by inter-story translation only, in x-direction} \\
iyF' &= \text{load induced in wall by inter-story translation only, in y-direction} \\
ix"F &= \text{load induced in wall by inter-story torsion only, in x-direction} \\
iy"F &= \text{load induced in wall by inter-story torsion only, in y-direction} \\
ixF &= \text{total external load to be resisted by a wall, in x-direction} \\
iyF &= \text{total external load to be resisted by a wall, in y-direction}
\end{align*}
\]

where:

- $F'_{ix}$ = load induced in wall by inter-story translation only, in x-direction
- $F'_{iy}$ = load induced in wall by inter-story translation only, in y-direction
- $F"_{ix}$ = load induced in wall by inter-story torsion only, in x-direction
- $F"_{iy}$ = load induced in wall by inter-story torsion only, in y-direction
- $F_{ix}$ = total external load to be resisted by a wall, in x-direction
- $F_{iy}$ = total external load to be resisted by a wall, in y-direction

To obtain $F'_{ix}$ and $F'_{iy}$, the forces $F_x$ and $F_y$ are distributed to the individual shear walls in proportion to their rigidities.
The force resisted by wall $i$ due to inter-story translation, in $x$-direction, is given by

$$F_{ix}' = \frac{F_x I_{iy}}{\sum I_{iy}}$$

The force resisted by wall $i$ due to inter-story translation, in $y$-direction, is given by

$$F_{iy}' = \frac{F_y I_{ix}}{\sum I_{ix}}$$

where:

- $F_x = \text{total external load to be resisted by all walls, in } x\text{-direction}$
- $F_y = \text{total external load to be resisted by all walls, in } y\text{-direction}$
- $I_{ix} = \text{second moment of area of a wall section about } x\text{ axis}$
- $I_{iy} = \text{second moment of area of a wall section about } y\text{ axis}$
- $\sum I_{ix} = \text{total second moment of areas of all walls in } x\text{-direction}$
- $\sum I_{iy} = \text{total second moment of area of all walls in } y\text{-direction}$

The force resisted by wall $i$ due to inter-sto ry torsion, in $x$-direction, is given by

$$F_{ix}'' = \frac{(F_x e_y) y_i I_{iy}}{\sum (x_i^2 I_{ix} + y_i^2 I_{iy})}$$

The force resisted by wall $i$ due to inter-sto ry torsion, in $y$-direction, is given by

$$F_{iy}'' = \frac{(F_y e_x) x_i I_{ix}}{\sum (x_i^2 I_{ix} + y_i^2 I_{iy})}$$

where:

- $x_i = \text{x-coordinate of a wall with respect to the center of rigidity C.R of the lateral load resisting system}$
- $y_i = \text{y-coordinate of a wall with respect to the center of rigidity C.R of the lateral load resisting system}$
- $e_x = \text{eccentricity resulting from non-coincidence of center of gravity C.G and center of rigidity C.R, in } x\text{-direction}$
- $e_y = \text{eccentricity resulting from non-coincidence of center of gravity C.G and center of rigidity C.R, in } y\text{-direction}$
Example (3): In Example (1), determine the forces acting on shear wall G.

Neglecting moments of inertia about weak axes, second moments of area of each of the shear walls about y-axis are given by

\[ I_{Ay} = I_{By} = I_{Gy} = I_{Hy} = \frac{0.2(3)^3}{12} = 0.45 \, m^4 \]

Total second moments of area about y-axis are given by

\[ \sum_{i=1}^{4} I_{iy} = 0.45(4) = 1.8 \, m^4 \]

Second moments of area of each of the shear walls about x-axis are given by

\[ I_{Cx} = I_{Dx} = I_{Ex} = I_{Fx} = \frac{0.2(3)^3}{12} = 0.45 \, m^4 \]

Total second moments of area about x-axis are given by

\[ \sum_{i=1}^{4} I_{ix} = 0.45(4) = 1.8 \, m^4 \]

To locate the center of rigidity C.R, the distance from the origin to the C.R \( \bar{y} \) in the y-direction is given by

\[ \bar{y} = \frac{\sum_{i=1}^{4} I_{iy} y_i}{\sum_{i=1}^{4} I_{iy}} = \frac{0.45(2)(18) + 2(0.45)(4.5)}{1.8} = 11.25 \, m \]

The distance from the origin to the C.R in the x-direction \( \bar{x} \) is given by
Thus, the eccentricity in y-direction \( e_y = 11.25 - 9.0 = 2.25 \text{ m} \)

And the eccentricity in x-direction \( e_x = 9.0 - 9.0 = 0.0 \text{ m} \)

Torsion caused by eccentricity \( e_y \), \( T_1 = 2.25 F_x \)

Torsion caused by accidental eccentricity, \( T_2 = F_x (0.05)(18) = 0.9 F_x \)

Total torsion, \( T_1 \pm T_2 = (2.25 F_x \pm 0.9 F_x) \)

\[
F'_{ix} = \frac{F_x I_{iy}}{\sum I_{iy}}
\]

\[
F'_{Ax} = F'_{Bx} = F'_{Gx} = F'_{Hx} = \frac{0.45 F_x}{1.8} = 0.25 F_x
\]

\[
F''_{ix} = \frac{(F_x e_y) y_j I_{iy}}{\sum (x_j^2 I_{ix} + y_j^2 I_{iy})}
\]

\[
F''_{Ax} = F''_{Bx} = F''_{Gx} = F''_{Hx} = \frac{(2.25 F_x \pm 0.9 F_x)(6.75)(0.45)}{2(0.45)(6.75)^2 + 2(0.45)(6.75)^2 + 2(0.45)(9)^2 + 2(0.45)(9)^2}
\]

\[
= 0.0133(2.25 F_x \pm 0.9 F_x)
\]

\[
= 0.042 F_x
\]

The forces acting on shear wall G are given by the following expression

\[
= 0.25 F_x + 0.042 F_x
\]

\[
= 0.292 F_x
\]

Using the story forces evaluated in Example (1), the forces acting on shear wall G at each of floor level are shown in the next figure:

![Distribution of forces at each floor level (Shear wall G)]
Classification of Structural Walls According To Seismic Risk

According to Chapters 2 and 21 of ACI 318-08, structural walls are defined as being walls proportioned to resist combinations of shears, moments, and axial forces induced by earthquake motions. A shear wall is a structural wall.

Reinforced concrete structural walls are categorized as follows:

1- **Ordinary reinforced concrete structural walls**: They are walls complying with the requirements of Chapters 1 through 18.

2- **Special reinforced concrete structural walls**: They are cast-in-place walls complying with the requirements of 21.2 and 21.7 in addition to the requirements for ordinary reinforced concrete structural walls.

Special Provisions for Earthquake Resistance

- According to Clause 1.1.9.1 of ACI 318-08, the seismic risk level of a region is regulated by the legally adopted general building code of which ACI 318-08 forms a part, or determined by local authority.

**Correlation between Seismic-Related Terminologies In Model Codes**

<table>
<thead>
<tr>
<th>Code/ Standard</th>
<th>Level of seismic risk as defined in the code section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low (21.1.2)</td>
</tr>
<tr>
<td>Uniform Building Code 1991, 1994, 1997</td>
<td>Seismic Zone 0, 1</td>
</tr>
</tbody>
</table>

SDC = Seismic Design Category

- According to Clauses 1.1.9.2 and 21.1.1.7 of ACI 318-08, in regions of low and intermediate seismic risk, provisions of Chapter 21 are not to be applied. (Chapter 1 through 18 are applicable)

- According to ACI 318-08, in regions of high seismic risk, special structural walls complying with 21.9 are to be used for resisting forces induced by earthquake motions.
Classification of Shear Walls According To Their Height-to-Length Ratios

Shear walls are classified as short or long according to their aspect ratios (the ratio of its height $h_w$ to length in the plane of loading $l_w$), as follows:

1- For $h_w/l_w < 2$, they are called short or squat shear walls. Their design is dominated by shear, rather than flexure. Aspect ratios below 2 mark the transition from slender to short behavior, and walls with such dimensions require considerable care in design if a ductile failure mode is required. Without this attention, shear walls are likely to fail in brittle failure modes such as diagonal tension or sliding shear rather than undergoing the more ductile flexural failure possible in slender walls. Short shear walls may need increased strength or special detailing, including diagonal steel to overcome these problems.

2- For $h_w/l_w \geq 2$, they are called long or slender shear walls. Their design is dominated by flexure. Aspect ratios are normally restricted to 7; higher ratios may result in inadequate stiffness, problems in anchoring the tension side of the shear wall and possibly significant amplifications due to $P-\Delta$ effects.

The above stated classification is not explicitly stated in ACI 318-08 Code.
Design of Ordinary Shear Walls

The shear wall is designed as a cantilever beam fixed at the base, to transfer load to the foundation. Shear forces, bending moments and axial loads are maximums at the base of the wall.

Types of Reinforcement:

To control cracking, shear reinforcement is required in the horizontal and vertical directions, to resist in plane shear forces.

The vertical reinforcement in the wall serves as flexural reinforcement. If large moment capacity is required, additional reinforcement can be placed at the ends of the wall within the section itself, or within enlargements at the ends. The heavily reinforced or enlarged sections are called boundary elements.

Shear Strength:

According to ACI 11.1.1, design of cross sections subject to shear are based on

\[ \Phi V_n \geq V_u \]  \hspace{1cm} (1)

where \( V_u \) is the factored force at the section considered and \( V_n \) is the nominal shear strength computed by

\[ V_n = V_c + V_s \]  \hspace{1cm} (2)

where \( V_c \) is nominal shear strength provided by concrete and \( V_s \) is nominal shear strength provided by shear reinforcement.
Based on ACI 11.9.3, \( V_{n,\text{max}} \) at any horizontal section for shear in plane of the wall is not to be taken greater than
\[
V_{n,\text{max}} = 2.65 \sqrt{f'_c} \ h \ d
\]
(3)
where \( h \) is thickness of wall, and \( d \) is the effective depth in the direction of bending, may be taken as 0.8\( l_w \), where \( l_w \) is length of wall considered in direction of shear force, as stated in ACI 11.9.4. A larger value of \( d \), equal to the distance from extreme compression fiber to center of force of all reinforcement in tension, be permitted to be used when determined by a strain compatibility analysis.

Based on ACI 11.9.5, the shear strength provided by concrete \( V_c \) is given by any of the following equations, as applicable.

For axial compression, Eqn. (4) is applicable
\[
V_c = 0.53 \sqrt{f'_c} \ h \ d
\]
(4)
For axial tension, Eqn. (5) is applicable
\[
V_c = 0.53 \left(1 - \frac{N_u}{35 A_g}\right) \sqrt{f'_c} \ h \ d
\]
(5)
where \( A_g \) is the gross area of wall section and \( N_u \) is the factored axial tension force in Eqn. (5).

ACI 11.9.6 specifies that a more detailed analysis is permitted to evaluate \( V_c \) as follows, where \( V_c \) is the lesser of the two values shown in Eqns. (6) and (7).

\[
V_c = 0.88 \sqrt{f'_c} \ h \ d + \frac{N_u \ d}{4 l_w}
\]
(6)
\[
V_c = \left[0.16 \sqrt{f'_c} + \frac{l_w \left(0.33 \sqrt{f'_c} + \frac{0.2 N_u}{l_w h}\right)}{M_u - l_w \ h V_c - \frac{l_w^2}{2}}\right] h \ d
\]
(7)
Where \( N_u \) is positive for compression and negative for tension. If \( \left(M_u / V_u - l_w / 2\right) \) is negative, Eqn. (7) is not applicable.
Shear Reinforcement:

A- When the factored shear force $V_u$ is less than $\Phi V_c / 2$, minimum wall reinforcement according to ACI 11.9.9 or in accordance with Chapter 14 of ACI code.

A-1 Minimum Horizontal Reinforcement Ratio:

Ratio of horizontal shear reinforcement area to gross concrete area of vertical section, $\rho_t$, shall not be less than 0.0025. Spacing of this reinforcement $S_2$ is not to exceed the smallest of $l_w / 5, 3 h, 45 \text{cm}$.

A-2 Minimum Vertical Reinforcement Ratio:

Ratio of vertical reinforcement area to gross concrete area of horizontal section, $\rho_t$ is not to be taken less than the larger of

$$\rho_t = 0.0025 + 0.50 \left( 2.5 - \frac{h_w}{l_w} \right) (\rho_t - 0.0025)$$

and 0.0025, but need not be greater than $\rho_t$ required by Eqn. (9). Spacing of this reinforcement $S_1$ is not to exceed the smallest of $l_w / 3, 3 h, 45 \text{cm}$.

Chapter 14 Provisions:

Minimum ratio of vertical reinforcement area to gross concrete area, $\rho_t$, shall be

- 0.0012 for deformed bars up to 16 mm in diameter, with $f_y$ not less than 4200 kg/cm².
- 0.0015 for other deformed bars.

Minimum ratio of horizontal reinforcement area to gross concrete area, $\rho_t$, shall be

- 0.0020 for deformed bars up to 16 mm in diameter, with $f_y$ not less than 4200 kg/cm².
- 0.0025 for other deformed bars.

B- When the factored shear force exceeds $\Phi V_c / 2$, minimum wall reinforcement for resisting shear, according to ACI 11.9.9, must be provided.

C- According to ACI 11.9.9.1 when the factored shear force $V_u$ exceeds $\Phi V_c$, horizontal shear reinforcement must be provided according to the following equation.

$$V_s = \frac{A_v f_y d}{S_2}$$

where $A_v$ is area of horizontal shear reinforcement within a distance $S_2$. Vertical shear reinforcement is provided using Eqn. (8), shown above.
The critical section for shear is taken at a distance equal to half the wall length \( l_w/2 \), or half the wall height \( h_w/2 \), whichever is less. Sections between the base of the wall and the critical section are to be designed for the shear at the critical section, as specified in ACI 11.9.7.

**Design for Flexure:**

The wall must be designed to resist the bending moment at the base and the axial force produced by the wall weight or the vertical loads it carries. Thus, it is considered as a beam-column.

For rectangular shear walls containing uniformly distributed vertical reinforcement and subjected to an axial load smaller than that producing balanced failure, the following equation, developed by Cardenas and Magura in ACI SP-36 in 1973, can be used to determine the approximate moment capacity of the wall.

\[
M_a = \Phi \left[ 0.5 A_s f_y l_w \left( 1 + \frac{P}{A_s f_y l_w} \right) \left( 1 - \frac{C}{l_w} \right) \right]
\]

Where:
\[
\frac{C}{l_w} = \frac{\omega + \alpha}{2\omega + 0.85\beta_i}
\]

\[
\omega = \frac{A_s f_y}{l_w h f'_c}
\quad \text{and} \quad
\alpha = \frac{P_u}{l_w h f'_c}
\]

\(C\) = distance from the extreme compression fiber to the neutral axis
\(A_s\) = total area of vertical reinforcement
\(l_w\) = horizontal length of wall
\(P_u\) = factored axial compressive load
\(f_y\) = yield strength of reinforcement
\(\Phi\) = strength reduction factor for bending

**Lateral Ties for Vertical Reinforcement:**

Based on ACI 14.3.6, vertical reinforcement need not be enclosed by lateral ties if vertical reinforcement is not greater than 0.01 times the gross concrete area, or where vertical reinforcement is not required as compression reinforcement.

**Additional Reinforcement around Openings:**

In addition to the required horizontal and vertical reinforcement explained earlier, ACI 14.3.7 states that not less than \(2\phi16\ mm\) bars are provided around all window and door openings. Such bars are to be extended to develop \(f_y\) in tension at the corners of the openings.

![Additional reinforcement around wall openings](image)