Design of T-Shaped Sections

Introduction

When floor slabs and their supporting beams are cast monolithically, they deflect along with the beams under the action of external loads. Therefore, slabs in the vicinity of the beams act as flanges for the beam. Interior beams have a flange on both sides, which are called T-beams. Edge beams have a flange on one side only, and referred to as L-beams as shown in Figure 8. Isolated T-beams, which are produced as precast concrete elements, are used in concrete construction.

Effective Flange Width

In the elementary theory of bending, stresses are assumed to be constant along the beam width. But if the flange width is very large, it is known that parts of the flanges at a distance from the web do not take their full share in resisting bending moment, and the stresses are varying.

To simplify analysis and design of T-sections, the actual stress distribution is replaced by uniform stress distribution based on the principle of static equivalence, shown in Figure 9.

According to ACI 8.12.2, effective flange width of a T-beam, $b_e$, shown in Figure 10, is not to exceed the smallest of:

- One-fourth the span length of the beam, $L/4$.
- Width of web plus 16 times slab thickness, $b_w + 16t$.
- Center-to-center spacing of beams.

where $b_w$ is the width of web, $t$ is the slab thickness, and $L$ is span of beam.

For L-shaped beams, ACI 8.12.3 requires that the effective flange width not to exceed the smallest of:
For isolated beams in which the T-shape is used to provide a flange for additional compression area, ACI 8.12.4 states that the flange thickness is not to exceed half the web width, $b_w/2$, and the effective flange width $b_e$ not more than four times the web width, $4b_w$.

**T- versus Rectangular Sections**

When T-shaped sections are subjected to negative bending moments, the flange is located in the tension zone. Since concrete strength in tension is usually neglected in strength design, the sections are treated as rectangular sections of width $b_w$. On the other hand, when sections are subjected to positive bending moments, the flange is located in the compression zone and the section is treated as a T-section shown in Figure 11.
Maximum Reinforcement, $A_{s,\text{max}}$

Strength of T and L-shaped sections is affected by the depth of the rectangular stress block, $a$. Two cases are considered, the first of which when the depth $a$ is smaller than, or equal to the flange thickness. The second case occurs when the depth $a$ is larger than the flange thickness.

**Case (I): when $a \leq t$**

The maximum amount of tension reinforcement, $A_{s,\text{max}}$ is evaluated when the net tensile strain in the extreme reinforcement is 0.005 and the strain at the extreme compression fiber is equal to 0.003, according ACI code 10.3.4.

The distance to the neutral axis, $x_{\text{max}}$ is given by the following equation and shown in Figure 12.

$$\frac{x_{\text{max}}}{d} = \frac{0.003}{0.003 + 0.005}$$

or,

$$x_{\text{max}} = 0.375 \ d$$

The resultant of the compressive forces in the concrete is given by

$$C_{\text{max}} = 0.85 \ f'_{c} \ a_{\text{max}} \ b_{e}$$

where

$$a_{\text{max}} = \beta_{J} x_{\text{max}}$$

The tensile force in the reinforcement is given by

$$T_{\text{max}} = A_{s,\text{max}} \ f_{y}$$

From equilibrium of forces in the axial direction,

$$T_{\text{max}} = C_{\text{max}}$$

but

$$A_{s,\text{max}} = \frac{T_{\text{max}}}{f_{y}}$$

The maximum reinforcement is given as

$$A_{s,\text{max}} = \frac{0.31875 \ f'_{c} \ \beta_{J} \ b_{e} \ d}{f_{y}}$$
Case (II) when $a > t$

The distance to the neutral axis, $x_{\text{max}}$, is given by the following equation and shown in Figure 13.

$$\frac{x_{\text{max}}}{d} = \frac{0.003}{0.003 + 0.005}, \text{ or}$$

$$x_{\text{max}} = 0.375 \, d$$

$$a_{\text{max}} = \beta_1 \, x_{\text{max}}$$

The compressive force in the concrete is divided into two forces, one represents the force in the rectangular beam $C_{1,\text{max}}$, and the second represents the forces in the flange overhangs $C_{2,\text{max}}$.

$$C_{1,\text{max}} = 0.85 \, f'_c \, a_{\text{max}} b_w$$

$$C_{2,\text{max}} = 0.85 \, f'_c \left(b_e - b_w\right) t$$

From equilibrium of forces,

$$T_{\text{max}} = C_{1,\text{max}} + C_{2,\text{max}}$$

$$T_{\text{max}} = 0.85 \, f'_c \left[\left(a_{\text{max}} b_w\right) + \left(b_e - b_w\right) t\right]$$

but $A_{s,\text{max}} = \frac{T_{\text{max}}}{f_y}$

$$A_{s,\text{max}} = \frac{0.85 \, f'_c \left[\beta_1 \, x_{\text{max}} b_w + \left(b_e - b_w\right) t\right]}{f_y}$$

Figure 13: Evaluation of maximum reinforcement; $a > t$
Minimum Reinforcement, $A_{r,min}$

According to ACI 10.5.2, for a statically determinate T-section with flanges in tension, the area shall not be less than the larger of the two following equations, with $b'_w$ is $2b_w$ or the width of flange, whichever is smaller.

$$A_{s,min} = \frac{0.80 \sqrt{f'_c}}{f_y} b'_w d$$

and

$$A_{s,min} = \frac{14}{f_y} b'_w d$$

Strength of T-sections

Case (I): when $a \leq t$

The problem starts with the assumption that $a \leq t$ as shown in Figure 14, then checking for the validity of this assumption once $a$ is evaluated.

The resultant of the compressive forces in the concrete is given by

$$C = 0.85 f'_c a b_e$$

The tensile force in the reinforcement is given by

$$T = A_s f_y$$

From equilibrium of forces in the axial direction,

$$T - C = 0$$

and

$$a = \frac{A_s f_y}{0.85 f'_c b_e}$$

![Figure 14: Forces in concrete and reinforcement; $a \leq t$](a)

If $a \leq t$ as assumed, then proceed on with the next step for evaluating the flexural capacity of the section. If not, quit case (I) and start the solution according to case (II), shown in the next section.
The nominal strength $M_n$ is evaluated using the equilibrium of moments acting on the section,

$$M_n = A_s f_y \left( \frac{d-a}{2} \right)$$

and the design moment is given by

$$M_d = \Phi A_s f_y \left( \frac{d-a}{2} \right)$$

**Case (II): when $a > t$**

The compressive force in the concrete is divided into two forces, one represents the force in the rectangular beam $C_1$, and the second represents the forces in the flange overhangs $C_2$, as shown in Figure 15.

**Figure 15: Forces in concrete and reinforcement; $a > t$**

$$C_1 = 0.85 f'_c a b_w$$

$$C_2 = 0.85 f'_c (b_e - b_w) t$$

From equilibrium of forces,

$$T = C_1 + C_2$$

From equilibrium of moments,

$$M_n = C_1 \left( d - \frac{a}{2} \right) + C_2 \left( d - \frac{t}{2} \right)$$

and the design moment is given by

$$M_n = \Phi \left[ C_1 \left( d - \frac{a}{2} \right) + C_2 \left( d - \frac{t}{2} \right) \right]$$

**Design of T-sections**

The design of a T-section involves the determination of five unknowns; $b_e$, $t$, $b_w$, $h$, and $A_s$. Material properties $f'_c$ and $f_y$ are specified by the structural designer. The slab thickness, $t$ is evaluated from the design process of the slab, a step that usually precedes the design of the beam. The effective flange width $b_e$, is evaluated according to **ACI 8.12.2 through 8.12.4**, given that spacing between adjacent beams, beam span, and slab thickness are known. The height of section $h$, is chosen to satisfy
both the strength requirement at the supports and the serviceability requirement according to ACI 9.5. The beam width, $b_n$, is chosen in such a way that it is enough for fitting the reinforcement with enough spacing. Therefore, the only quantity that needs to be determined is the area of reinforcement, $A_s$.

The design procedure is summarized as follows:

1. Maximum amount of reinforcement permitted by ACI Code, $A_{s,\text{max}}$ is determined.
2. Using the moment strength equation, the depth of the rectangular stress block in concrete, $a$ is determined through solving a quadratic equation in terms of $a$.
3. The depth $a$ is checked to see if it is smaller than or larger than $t$. If $a \leq t$, then the required reinforcement is evaluated from the equation
   \[ A_s = \frac{T}{f_y} \cdot a \]
   If $a > t$, then steps 2 and 3 are to be repeated using the moment strength equation
   \[ M_n = C_1 (d - a/2) + C_2 (d - t/2) \]
4. The evaluated reinforcement is checked against $A_{s,\text{max}}$ and $A_{s,\text{min}}$. If $A_s \leq A_{s,\text{max}}$, then the evaluated reinforcement is satisfactory, if not, the dimensions of the cross section are to be enlarged until this condition is satisfied.
5. Bar diameters and numbers are chosen, and the minimum clear spacing between bars is to be satisfied.
6. The cross section along with the required reinforcement is neatly drawn to an appropriate drawing scale.

**Strength of Non-Rectangular Sections**

For any section that is symmetrical with respect to a vertical axis, or any section that deflects vertically without twisting, Whitney’s rectangular compressive stress distribution may be used in accordance with ACI 10.2.7.
Doubly Reinforced Rectangular Sections

Introduction

Beams containing steel reinforcement at the tension and compression sides are called doubly reinforced sections. Doubly reinforced sections are useful in case of singly-reinforced sections being unable to provide the required bending strength, even when maximum reinforcement ratio is used. Steel on the compression side increases the moment capacity of a given section, reduces long-term deflection, improves ductility and eases fixing of reinforcement.

![Diagram](image)

**Figure 16:** Doubly reinforced section: (a) cross section; (b) strains; (c) forces

Strength of Doubly Reinforced Rectangular Sections

For doubly-reinforced sections, two possible situations are possible depending on the compression reinforcement, $A'_s$.

a- Compression reinforcement, $A'_s$ yields ($\varepsilon'_c \geq \varepsilon_y$):

The resultant of the compressive forces in the concrete is given by

$$C_c = 0.85 f'_c a b$$

The compressive force in the compression steel is given by

$$C_s = A'_s (f_y - 0.85 f'_c)$$

The force in the tension steel is given by (assuming $\varepsilon_r \geq \varepsilon_y$)

$$T = A_s f_y$$

The first of the equilibrium equations is written as

$$T = C_c + C_s$$

and $a$ is evaluated as
\[
a = \frac{f_y (A_s - A'_s) + 0.85 A'_s f'_c}{0.85 f'_c b}
\]

The second equilibrium equation is given as

\[
M_a = C_c (d - a / 2) + C_s (d - d')
\]

where \(d'\) distance from extreme compression fiber to centroid of longitudinal compression reinforcement.

and the design moment, \(M_d\) is given as

\[
M_d = \Phi \left[ C_c (d - a / 2) + C_s (d - d') \right]
\]

**b- Compression reinforcement, \(A'_s\) doesn't yield \((\varepsilon'_s < \varepsilon_y)\):**

The resultant of the compressive forces in the concrete is given by

\[
C_c = 0.85 f'_c a b
\]

The compressive force in the compression steel is given by

\[
C_s = A'_s \left( f'_s - 0.85 f'_c \right)
\]

The strain in the compression reinforcement \(\varepsilon'_s\) is evaluated from the following expression

\[
\frac{\varepsilon'_s}{0.003} = \frac{x - d'}{x}
\]

where \(f'_s = \varepsilon'_s E_s\)

The force in the tension steel is given by (assuming \(\varepsilon_t \geq \varepsilon_y\))

\[
T = A_s f_y
\]

The first of the equilibrium equations is written as

\[
T = C_c + C_s
\]

and \(a\) is evaluated as one of the logic roots of a quadratic equation in terms of \(a\).

The second equilibrium equation is given as

\[
M_a = C_c (d - a / 2) + C_s (d - d')
\]

and the design moment, \(M_d\) is given as

\[
M_d = \Phi \left[ C_c (d - a / 2) + C_s (d - d') \right]
\]
**Summary of Moment Capacity Procedure**

1. Assuming case (a) where the compression reinforcement yields ($\varepsilon_s' \geq \varepsilon_y$), evaluate $a$.

2. Check the validity of the assumption $\varepsilon_s' \geq \varepsilon_y$. If assumption is valid, evaluate the strength reduction factor, $\Phi$. If assumption is invalid skip go directly to steps 4 and 5.

3. The moment capacity is evaluated from the equation
   \[ M_d = \Phi \left[ C_c \left( d - a / 2 \right) + C_s \left( d - d' \right) \right] \]

4. Following case (b), evaluate $x$ and $\Phi$.

5. The moment capacity is evaluated from the equation
   \[ M_d = \Phi \left[ C_c \left( d - a / 2 \right) + C_s \left( d - d' \right) \right] \]

**Summary of Design Procedure**

1. Determine the design moment ($\phi M_{nI}$) that the beam can carry using maximum amount of tension reinforcement, as a tension-controlled section.

2. If $\phi M_{nI}$ is larger in magnitude than the total factored moment $M_u$, then the section is to be designed as singly reinforced (dealt with before).

3. If $\phi M_{nI}$ is smaller than $M_u$, then the section is to be designed as doubly reinforced. The remaining moment, $M_u - \phi M_{nI}$ is to be resisted by a couple resulting from forces in compression steel and the additional tension steel.

4. The magnitude of the additional tensile force in the reinforcement is evaluated as
   \[ T_2 = \frac{M_u - \phi M_{nI}}{d - d'} \]

5. Determine the area of additional tension reinforcement, $A_{s2} = \frac{T_2}{f_y}$.

5. Determine the total area of tension reinforcement, $A_s = A_{s, max} + A_{s2}$

6. Determine the area of compression reinforcement as
   \[ A'_{s} = T_2 \left( \frac{f_y}{f'c} \right) \]

7. Check the moment strength of the cross section.