Design of Raft Footings

The structural design of raft foundations can be carried out by two methods; the conventional rigid method and the approximate flexible method. In this section, only the rigid method will be covered.

Design of raft footings is detailed in the following steps.

1- Select a trial footing depth:

According to ACI Code 15.7, depth of footing above reinforcement is not to be less than 15 cm for footings on soil. Noting that 7.5 cm of clear concrete cover is required if concrete is cast against soil, a practical minimum depth is taken as 25 cm.

2- Determine the allowable pressure \( q \) on the soil below the raft:

Determine the allowable soil pressure below the raft at its corner points and check whether the pressure values are less than the net allowable soil pressure using the following equation

\[
q = \sum \frac{P}{A} \pm \frac{M_{x,y}}{I_x} \pm \frac{M_{x,x}}{I_y}
\]

where

\[
I_x = B \frac{L^3}{12} = \text{moment of inertia about the x-axis}
\]
\[ I_y = \frac{LB^3}{12} \] = moment of inertia about the y-axis

\[ M_x = \sum P e_x \] = moment of the column loads about the x-axis

\[ M_y = \sum P e_y \] = moment of the column loads about the y-axis

\( e_x \) and \( e_y \) are the load eccentricities in the directions of x and y. These load eccentricities can be determined using X, Y coordinates

\[
X_R = \frac{P_1 X_1 + P_2 X_2 + P_3 X_3 + \ldots}{\sum P} \quad \text{and} \quad e_x = X_R - X_g
\]

\[
Y_R = \frac{P_1 Y_1 + P_2 Y_2 + P_3 Y_3 + \ldots}{\sum P} \quad \text{and} \quad e_y = Y_R - Y_g
\]

where \( X_g \) and \( Y_g \) are coordinates of center of gravity of raft measured with respect to X and Y coordinates, \( X_R \) and \( Y_R \) are coordinates of resultant of loads, \( c.g \) is center of gravity, and \( c.l. \) is center of loads as shown in Figure 11.19.a.

3- **Compute the net factored soil pressure under the raft, as shown in Figure 11.19.b**

![Figure 11.19.b: Net factored soil pressure](image)

4- **Check footing depth for punching shear around columns.**

5- **Divide the raft into individual strips between column lines in x and y directions, as shown in Figure 11.19.a and Figure 11.19.c.**
6- **Draw S.F.D and B.M.D for each individual strip:**

For example for the vertical strip in the y-direction shown in Figure 11.19.a and 11.19.b, which width is equal to \( B \), the total soil reaction is equal to \( \frac{q_{\text{min}} + q_{\text{max}}}{2} B_i L \), where \( q_{\text{min}} \) is the average pressure value at one edge of the strip, and \( q_{\text{max}} \) is the average pressure value at the other edge of the strip. The total column load on this strip is \( P_{1a} + P_{4a} + P_{7a} + P_{10a} + P_{13a} \). The sum of the column loads on the strip will not be equal to the total soil reaction due to presence of shear stresses between every neighboring strips. Therefore, the soil reaction and the column loads need to be modified on an average load basis. The average load is the sum of the total column loads and total soil reaction divided by 2.0. Modified minimum soil pressure \( q_{\text{min,m}} \) at one edge of the strip is given by

\[
q_{\text{min,m}} = \frac{q_{\text{min}} \text{(average load)}}{q_{\text{min}} + q_{\text{max}}/2} B_i L
\]

The modified maximum soil pressure at the other edge of the strip is given by

\[
q_{\text{max,m}} = \frac{q_{\text{max}} \text{(average load)}}{q_{\text{min}} + q_{\text{max}}/2} B_i L
\]
The column load modification factor \( CF \) that is to be multiplied by each column load is given as

\[
CF = \frac{\text{average load}}{P_{1a} + P_{4a} + P_{7a} + P_{10a} + P_{13a}}
\]

If the loads and the spans are symmetrical, the soil pressure under the strip is taken uniform by considering the average of the soil pressure at two ends of the strip.

At this time, the shear and moment diagrams for this strip, using modified loads can be drawn. This process is to be repeated for all strips in the x and y directions.

7- Check footing depth for beam shear for each of the individual strips.

8- Determine the required amount of reinforcement for each of the strips.

9- Check bearing strength under the columns.

10- Check reinforcing bars for anchorage lengths.

11- Prepare neat design drawings showing footing dimensions and provided reinforcement.

Example (11.10):

Design a raft footing to support twelve columns \( C1 \) through \( C12 \) as shown in Figure 11.20.a. All columns are \( 40 \, \text{cm} \times 40 \, \text{cm} \) in cross section. Column loads are shown in Table 11.1.

Use \( f'_{c} = 350 \, \text{kg} / \text{cm}^2 \), \( f'_{y} = 4200 \, \text{kg} / \text{cm}^2 \), and \( q_{all} (\text{net}) = 1.0 \, \text{kg} / \text{cm}^2 \).

Solution:

1- Select a trial footing depth:

Let footing thickness be equal to \( 50 \, \text{cm} \).
2- Determine the allowable pressure \( q \) on the soil below the raft:

From Table 11.1, load eccentricities are calculated as shown below.

<table>
<thead>
<tr>
<th>No.</th>
<th>( P_{D} ) (ton)</th>
<th>( P_{L} ) (ton)</th>
<th>Ser. Load</th>
<th>Fac. Load</th>
<th>( X_{i} ) (m)</th>
<th>( Y_{i} ) (m)</th>
<th>( P_{X} ) (ser.)</th>
<th>( P_{Y} ) (ser.)</th>
<th>( P_{X} ) (fac.)</th>
<th>( P_{Y} ) (fac.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>20</td>
<td>20</td>
<td>40</td>
<td>56</td>
<td>0.5</td>
<td>12.5</td>
<td>20</td>
<td>500</td>
<td>28</td>
<td>700</td>
</tr>
<tr>
<td>C2</td>
<td>30</td>
<td>20</td>
<td>50</td>
<td>68</td>
<td>5.5</td>
<td>12.5</td>
<td>275</td>
<td>625</td>
<td>374</td>
<td>850</td>
</tr>
<tr>
<td>C3</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>42</td>
<td>9.5</td>
<td>12.5</td>
<td>285</td>
<td>375</td>
<td>399</td>
<td>525</td>
</tr>
<tr>
<td>C4</td>
<td>80</td>
<td>40</td>
<td>120</td>
<td>160</td>
<td>0.5</td>
<td>8.5</td>
<td>60</td>
<td>1020</td>
<td>80</td>
<td>1360</td>
</tr>
<tr>
<td>C5</td>
<td>100</td>
<td>50</td>
<td>150</td>
<td>200</td>
<td>5.5</td>
<td>8.5</td>
<td>825</td>
<td>1275</td>
<td>1100</td>
<td>1700</td>
</tr>
<tr>
<td>C6</td>
<td>60</td>
<td>30</td>
<td>90</td>
<td>120</td>
<td>9.5</td>
<td>8.5</td>
<td>855</td>
<td>765</td>
<td>1110</td>
<td>1020</td>
</tr>
<tr>
<td>C7</td>
<td>80</td>
<td>40</td>
<td>120</td>
<td>160</td>
<td>0.5</td>
<td>4.5</td>
<td>60</td>
<td>540</td>
<td>80</td>
<td>720</td>
</tr>
<tr>
<td>C8</td>
<td>100</td>
<td>50</td>
<td>150</td>
<td>200</td>
<td>5.5</td>
<td>4.5</td>
<td>825</td>
<td>675</td>
<td>1100</td>
<td>900</td>
</tr>
<tr>
<td>C9</td>
<td>60</td>
<td>30</td>
<td>90</td>
<td>120</td>
<td>9.5</td>
<td>4.5</td>
<td>855</td>
<td>405</td>
<td>1140</td>
<td>540</td>
</tr>
<tr>
<td>C10</td>
<td>20</td>
<td>20</td>
<td>40</td>
<td>56</td>
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<td>0.5</td>
<td>20</td>
<td>20</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>C11</td>
<td>30</td>
<td>20</td>
<td>50</td>
<td>68</td>
<td>5.5</td>
<td>0.5</td>
<td>275</td>
<td>25</td>
<td>374</td>
<td>34</td>
</tr>
<tr>
<td>C12</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>42</td>
<td>9.5</td>
<td>0.5</td>
<td>285</td>
<td>15</td>
<td>399</td>
<td>21</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td></td>
<td></td>
<td>960</td>
<td>1292</td>
<td>4640</td>
<td>6240</td>
<td>6242</td>
<td>8398</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For service loads,
For service loads,

\[ q = \frac{P}{A} \pm \frac{M_y}{I_y} x \]

\[ q = \frac{960}{13(10)} \pm \frac{960(0.17)x}{13(10)^3/12} \]

\[ q_A = q_D = q_{max} = \frac{960}{13(10)} + \frac{960(0.17)(5)}{13(10)^3/12} = 8.14 \text{ t/m}^2 < 10 \text{ t/m}^2 \]

i.e. footing is adequate for supporting the applied loads safely without causing shear failure in the soil.

3- Compute the net factored soil pressure under the raft:

\[ X_R = \frac{\sum P_i X_i}{\sum P_i} = \frac{6242}{1292} = 4.83 \text{ m}, \quad Y_R = \frac{\sum P_i Y_i}{\sum P_i} = \frac{8398}{1292} = 6.50 \text{ m} \]

\[ e_x = 5.0 - 4.83 = 0.17 \text{ m} \]

\[ e_y = 6.5 - 6.5 = 0.0 \]

\[ q_u = \frac{P}{A} \pm \frac{M_y}{I_y} x \]

\[ q_u = \frac{1292}{13(10)} \pm \frac{1292(0.17)x}{13(10)^3/12} \]
Figure 11.20.b: Factored soil pressure.

\[ q_u = 9.94 \pm 0.20 \times \]

\[ q_A = q_D = 9.94 + 0.2(5) = 10.94 \, t/m^2 \]

\[ q_B = q_C = 9.94 - 0.2(5) = 8.94 \, t/m^2 \]

The pressures are shown in Figure 11.20.b.

4- Check footing depth for punching shear around columns:

Perimeter of punching shear surfaces is the same for corner, side, and interior columns. Since the largest load is supported by interior columns (C5 and C8), only punching shear under interior columns will be checked. The sections for punching shear are shown in Figure 11.20.c.
Average effective depth \( d = 50 - 7.5 - 2.0 = 40.5 \, \text{cm} \)

The factored punching shear force \( V_u \) is given by:

\[
V_u = 200 \left( \frac{9.92 + 9.76}{2} \right) (0.805)(0.805) = 193.6 \, \text{tons}
\]

\( b_o = 4(40+40.5) = 322.0 \, \text{cm} \)

\[
\Phi V_c = \lambda \Phi \sqrt{f'c} b_o d
\]

\[
= 0.75 \sqrt{350} (322)(40.5)/1000 = 182.98 \, \text{tons} < 193.60 \, \text{tons}
\]

Increase footing thickness to 55 cm to satisfy punching shear requirement.

Average effective depth \( d = 55 - 7.5 - 2.0 = 45.5 \, \text{cm} \)

\( b_c = 4(40+45.5) = 342.0 \, \text{cm} \)

\[
\Phi V_c = \lambda \Phi \sqrt{f'c} b_o d
\]

\[
= 0.75 \sqrt{350} (342)(45.5)/1000 = 218.34 \, \text{tons} > 200 \, \text{tons}
\]

5- Divide the raft into individual strips in X and Y directions:

The raft is divided into seven strips; three in the Y-direction, and four in the X-direction, as shown in Figure 11.20.b and Figure 11.20.d.

6- Draw S.F.D and B.M.D for the strip:

\[ \text{Figure 11.20.d: Strips in X direction} \]

\[ \text{Design of strip AGHD:} \]

\[ 6- \text{ Draw S.F.D and B.M.D for the strip:} \]
\[ q_{\text{avg}} = \frac{10.94 + 10.34}{2} = 10.64 \text{ t/m}^2 \]

\[ w_t = 10.64 \times 3 = 31.92 \text{ t/m} \]

Total column load on the strip = 56(2) + 160(2) = 432 tons

Total soil reaction = 31.92(13) = 414.96 tons

Average load = (432 + 414.96)/2 = 423.84 tons

Column modification factor = 423.84/432 = 0.9811

Soil pressure modification factor = 423.84/414.96 = 1.0214

Modified soil pressure = (1.0214)(31.92) = 32.60 t/m

---

**Figure 11.20 e:** (1) Loading; (2) modified loading; (3) shear force diagram; (4) bending moment diagram

**7-** Check footing depth for beam shear for the strip:

\[ \Phi V_c = 0.75(0.53)\sqrt{350(300)(45.5)/1000} = 101.51 \text{ tons} > 91.76 \text{ O.K.} \]

**8-** Determine the required amounts of reinforcement for the strip:

To evaluate the reinforcement at the section of maximum positive moment
Design of strips DCIJ and ABMN:

6- Draw S.F.D and B.M.D for each of the two strips:

Total column load on the strip = 56 + 68 + 42 = 166 tons
Total soil reaction = (27.35 + 22.35) (10/2) = 248.5 tons
Average load = (166 + 248.5)/2 = 207.25 tons
Column modification factor = 207.25/166 = 1.2485
Soil pressure modification factor = 207.25/248.5 = 0.8340
On one edge of strip, soil pressure $w_u = 22.35 (0.8340) = 18.64 t/m$
On the other edge of strip, soil pressure $w_u$ is given by $w_u = 27.35 (0.8340) = 22.81 t/m$

Soil load under footing is given by the following expression
$q(x) = 22.81 - 0.417 x$, where $x$ is measured from the left side of the strip
The shear force $V(x) = 22.81 x - 0.2085 x^2 + Const.$
The bending moment $M(x) = 11.405 x^2 - 0.0695 x^3 + Const.$

Zero-shear points are evaluated by equating the expression for shear to zero
($x = 3.17 m$, and $x = 7.27 m$).
Figure 11.20.g: (1) Modified loading; (2) shear force diagram; (3) bending moment diagram

7- Check footing depth for beam shear for each of the two strips:

\[ \Phi V_c = 0.75(0.53)\sqrt{350\,(250)\,(45.5)}/1000 = 84.59 \text{ tons} \quad > V_{u,\text{max}} \]

8- Determine the required amounts of reinforcement for each of the two strips:

To evaluate the reinforcement at the section of maximum negative moment

\[ \rho = \frac{0.85(350)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5 \,(74.29)}{0.9\,(250)\,(45.5)^2\,(350)}} \right] = 0.003905 \]

\[ A_s = 0.003905(250)(45.5) = 44.42 \text{ cm}^2 \], use \( \phi \) 16 mm @ 10 cm

For \( M_u = 51.57 \text{ t.m} \)

\[ \rho = \frac{0.85(350)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5 \,(47.55)}{0.9\,(250)\,(45.5)^2\,(350)}} \right] = 0.00247 \]
\[ A_s = 0.00247(250)(45.5) = 28.1 \text{cm}^2 \], use \( \phi \ 14 \text{ mm} \ @ \ 10 \text{ cm} \)

At the section of maximum positive moment, \( \rho < \rho_{\text{min}} \), so use \( \rho = \rho_{\text{min}} = 0.0018 \)

\[ A_s = 0.0018(250)(55) = 24.75 \text{cm}^2 \], use \( \phi \ 12 \text{ mm} \ @ \ 10 \text{ cm} \)

For reinforcement details, see Figure 11.20.h.

Figure 11.20.h: Reinforcement in strips \( DCIJ \) and \( ABMN \)

6- Prepare neat design drawings, showing footing dimensions and provided reinforcement:

Design drawings are shown in Figures 11.20.g and 11.20.h.