Design of Strap (Cantilever) Footings

This is a special type of footing used for two columns. The two columns are provided by two separate footings connected by a rigid beam called “strap beam”. The footing areas are proportioned in such away to keep the pressure under the two footings uniform and equal and for the centroid of the combined footing areas to coincide with the resultant of the two column loads. It is assumed that the strap beam is rigid and does not transfer any load by bearing on the soil at its bottom contact surface.

Summary of strap footing design is shown in the following steps.

1- Select trial footing depths.
2- Proportion footing dimensions.
3- Evaluate factored net soil pressure under the footings.
4- Design column footings for beam shear and moment.
5- Design the strap beam for moment and shear.
6- Check bearing strength of column and footing concrete.
7- Check chosen reinforcement bars for anchorage.
8- Prepare detailed design drawings.

Example (11.9):

Design a strap footing to support two columns, shown in Figure 11.18.a, spaced at a distance of 6.0 m center-to-center. Column A is 40 cm × 40 cm and carries a dead load of 50 tons and a live load of 30 tons. Column B is also 40 cm × 40 cm in cross section and carries a dead load of 70 tons and a live load of 50 tons.

Use $f'_c = 250 \text{ kg} / \text{cm}^2$, $f_y = 4200 \text{ kg} / \text{cm}^2$, and $q_{all \ (net)} = 1.5 \text{ kg} / \text{cm}^2$. 
Solution:

1- Select trial footing depths:
Assume footing thickness of 40 cm.

2- Proportion footing dimensions:

\[
A_{\text{req}} = \frac{P_A + P_B}{q_{\text{all}} (\text{net})}
\]

\[
A_{\text{req}} = \frac{50 + 30 + 70 + 50}{15.0} = 13.33 \text{ m}^2
\]

\[
B L_1 + B L_2 = 13.33 \text{ m}^2
\]

The distance from the resultant of the column forces to the center of column B is given as:

\[
\bar{x} = \frac{50 + 30)(6)}{200} = 2.4 \text{ m}
\]

Center of gravity of the two footings should coincide with the resultant of the two column loads to ensure uniform soil pressure below the two footings.

Taking moments of footing areas about the center of column B,

\[
A_A (6 + 0.2 - L_1 / 2) = (A_A + A_B) \bar{x}
\]

\[
B L_1 (6.2 - L_1 / 2) = (B L_1 + B L_2) \bar{x}
\]

\[
\bar{x} = \frac{B L_1 (6.2 - L_1 / 2)}{(B L_1 + B L_2)} = \frac{B L_1 (6.2 - L_1 / 2)}{13.33}
\]
\[ \bar{x} = 2.4 = \frac{B L \left( 6.2 - L / 2 \right)}{13.33} \]

Let \( B = 2.5 \, m \),
\[ 2.4 = \frac{2.5 L \left( 6.2 - L / 2 \right)}{13.33} \]
\[ L^2 / 2 - 6.2 L + 12.8 = 0 \]
Solving this quadratic equation gives
\( L_1=2.62 \, m \), or \( L_2=9.78 \, m \) (rejected).
Use \( L_1=2.60 \, m \), and \( L_2=2.75 \, m \)

3- Evaluate factored net soil pressure under the footings:
\[ q_u (net) = \frac{P_{Au} + P_{Bu}}{B(L_1 + L_2)} = \frac{1.2(50+70)+1.6(30+50)}{(2.6 + 2.75)(2.5)} = 20.34 \, t / m^2 \]

4- Design column footings for beam shear and moment:

4-1 Beam shear:
Effective depth \( d = 40 - 7.5 - 0.9 = 31.6 \, cm \) (lower layer)

a- Exterior footing:
\[ V_u = 20.34 \left( 2.6 \right) \left[ \frac{2.5 - 0.4}{2} \right] - 0.316 = 38.82 \, tons \]
\[ \Phi V_c = 0.75 \left( 0.53 \right) \sqrt{250 \left( 260 \right) \left( 31.6 \right) / 1000} = 51.64 \, tons > 38.82 \, tons \]

b- Interior footing:
\[ V_u = 20.34 \left( 2.75 \right) \left[ \frac{2.5 - 0.4}{2} \right] - 0.316 = 41.06 \, tons \]
\[ \Phi V_c = 0.75 \left( 0.53 \right) \sqrt{250 \left( 275 \right) \left( 31.6 \right) / 1000} = 54.62 \, tons > 41.06 \, tons \]
Punching shear need not be considered due to presence of the strap beam.

Figure 11.18.b: Critical sections for beam shear
4-2 Bending moment:

a- Exterior footing:

\[ M_u = 20.34 \left( \frac{2.6}{2} \right) \left( \frac{2.5-0.4}{2} \right)^2 = 29.15 \text{ t.m} \]

\[ \rho = \frac{0.85(250)}{4200} \left[ 1 - \frac{1 - 2.353(10)^5 \left( \frac{29.15}{0.9(260)(31.6)^2(250)} \right)}{} \right] = 0.003063 \]

\[ A_s = 0.003063(260)(31.6) = 25.17 \text{ cm}^2, \text{ use } 13\phi 16 \text{ mm} \]

b- Interior footing:

\[ M_u = 20.34 \left( \frac{2.75}{2} \right) \left( \frac{2.5-0.4}{2} \right)^2 = 30.83 \text{ t.m} \]

\[ \rho = \frac{0.85(250)}{4200} \left[ 1 - \frac{1 - 2.61(10)^5 \left( \frac{30.83}{0.9(275)(31.6)^2(250)} \right)}{} \right] = 0.003063 \]

\[ A_s = 0.003063(275)(31.6) = 26.62 \text{ cm}^2, \text{ use } 14\phi 16 \text{ mm} \]

For shrinkage reinforcement in longitudinal direction,

\[ A_s = 0.0018(250)(40) = 18.0 \text{ cm}^2, \text{ use } 16\phi 12 \text{ mm} \]

![Figure 11.18.c: Footing reinforcement](image)

5- Design of strap beam for moment and shear:

5-1 For moment:

The strap beam shall be proportioned for a maximum negative moment of 92.90 t.m, shown in Figure 11.18.f.

Assume a cross section of 40 cm × 100 cm for the strap beam

\[ d_{ve} = 100 - 7.5 - 1.0 - 2.0 - 1.25 = 88.25 \text{ cm} \]
\[ \rho_{-ve} = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5(92.9)}{0.9(40)(88.25)^2(250)}} \right] = 0.00862 \]

\[ A_{s-ve} = 0.00862(40)(88.25) = 30.43 \text{ cm}^2, \text{ use 8\# 22 mm} \]

\[ d = 100 - 7.5 - 1.6 - 0.7 - 1.0 = 89.2 \text{ cm} \]

\[ \rho_{+ve} = \frac{0.85(250)}{4200} \left[ 1 - \sqrt{1 - \frac{2.353(10)^5(48.07)}{0.9(40)(89.2)^2(250)}} \right] = 0.004167 \]

\[ A_{s+ve} = 0.004167(40)(90.2) = 14.87 \text{ cm}^2, \text{ use 8\# 16 mm} \]

Figure 11.18: (continued); (d) Loading diagram; (e) shearing force diagram; (f) bending moment diagram
5-2 For shear:
The shearing force diagram is shown in Figure 11.18.e.
Critical shear at distance 88.25 cm from face of column A (to the right) is:

\[ V_u = 42.67 \text{ tons} \]

\[ \Phi V_c = 0.75 \left( 0.53 \right) \sqrt{250 \left( 40 \right) \left( 88.25 \right)} / 1000 = 22.19 \text{ tons} \]

\[ V_s = \frac{\left( 42.67 - 22.19 \right)}{0.75} = 27.31 \text{ tons} \]

\[ \left( \frac{A_v}{S} \right) = \frac{27.31 \left( 1000 \right)}{88.25 \left( 4200 \right)} = 0.074 \text{ cm}^2 / \text{cm} > \frac{3.5 \left( 40 \right)}{4200} \text{ cm}^2 / \text{cm} \]

Try \( \Phi 10 \text{ mm} \) stirrups

\[ \frac{2 \left( 0.785 \right)}{S} = 0.074 \text{ cm}^2 / \text{cm} \]

Use \( \Phi 10 \text{ mm} \) stirrups @ 20 cm.

6- Check bearing strength of column and footing concrete:

\[ \Phi P_n = 0.65 \left( 0.85 \right) \left( 250 \right) \left( 40 \right) / 1000 = 221 \text{ tons} > 164 \text{ tons} \]

i.e. use minimum dowel reinforcement, \( A_s = 0.005 \left( 40 \right) \left( 40 \right) = 8.0 \text{ cm}^2 \)

7- Check chosen reinforcing bars for anchorage:

a- Footing reinforcement (\( \Phi 16 \text{ mm} \)):

\[ \psi_e = \psi_s = \lambda = 1 \text{ and } \psi_s = 0.8 \]

\( c_b \) is the smaller of:

\[ 7.5 + 0.8 = 8.3 \text{ cm}, \text{ or } c_b = \frac{260 - 15 - 1.6}{14 \left( 2 \right)} = 8.69 \text{ cm}, \text{ i.e., } c_b = 8.3 \text{ cm} \]

\[ \frac{c_b + K_{te}}{d_b} = \frac{8.3 + 0}{1.6} = 5.19 > 2.5, \text{ take it equal to } 2.5 \]

\[ l_d = \frac{1.6 \left( 0.8 \right) \left( 4200 \right)}{3.5 \left( 2.5 \right) \sqrt{250}} = 38.86 \text{ cm} \]

Available development length = 105.0 - 7.5 = 97.5 cm > 38.86 cm

b- Bottom reinforcement in strap beam (\( \Phi 16 \text{ mm} \)):

\[ \psi_e = \psi_s = \lambda = 1 \text{ and } \psi_s = 0.8 \]

\( c_b \) is the smaller of:
7.5 + 1.0 + 0.8 = 9.3 cm, or \( c_b = \frac{40 - 15 - 2 - 1.6}{5(2)} = 2.14 \) cm, i.e., \( c_b = 2.14 \) cm

\[
\frac{c_b + K_{tr}}{d_b} = \frac{2.14 + 0}{1.6} = 1.34
\]

\[
l_d = \frac{1.6(0.8)(4200)}{3.5(1.34)\sqrt{250}} = 72.50 \text{ cm}
\]

Available development length = 117.5 – 7.5 = 110.0 cm > 72.50 cm

**c- Top reinforcement in strap beam (ø 22 mm):**

\( \psi_t = 1.3 \) since more than 30 cm of fresh concrete is cast below the reinforcement, \( \psi_e = \psi_x = \lambda = 1 \)

\( c_b \) is the smaller of:

7.5 + 1.0 + 1.1 = 9.6 cm, or \( c_b = \frac{40 - 15 - 2 - 2.2}{4(2)} = 2.6 \) cm, i.e., \( c_b = 2.6 \) cm

\[
\frac{c_b + K_{tr}}{d_b} = \frac{2.6 + 0}{2.2} = 1.18
\]

\[
l_d = \frac{(1.3)(2.2)(4200)}{3.5(1.18)\sqrt{250}} = 183.95 \text{ cm}
\]

Available development length = 213.0 – 7.5 = 205.5 cm > 183.95 cm

**8- Prepare detailed design drawings:**

Detailed design drawings are shown in Figure 11.18.c and Figure 11.18.g.

![Design drawings](image-url)