2.11. A wooden block is 2 m long, 1 m wide, and 1 m deep. What is the specific gravity of the block if the metacenter is at the same point as the center of gravity? Is the floating block stable? Explain.

2.12. A 12-m-long, 4.8-m-wide, and 4.2-m-deep rectangular pontoon has a draft of 2 m in seawater (sp. gr. = 1.03). Assuming the load is uniformly distributed on the bottom of the pontoon, and the maximum design angle of heel is 15°, determine the distance that the center of gravity can be moved from the center line towards the edge of the pontoon.

3.1 Description of Pipe Flow

In hydraulics, the term pipe flow usually refers to full water flow in closed conduits of circular cross sections under a certain pressure gradient. For a given discharge \( Q \), the pipe flow at any section of the pipe can be described by the pipe cross section, the pipe elevation, the pressure, and the flow velocity in the pipe.

The elevation \( h \) of a particular section in the pipe is usually measured with respect to a horizontal reference datum, such as the mean sea level (MSL) at a certain estuary. The pressure in a pipe generally varies from one point to another, but a mean value is normally used over a particular section of interest. In other words, the regional pressure variation in a given cross section is commonly neglected unless otherwise specified.

In most engineering computations the section mean velocity \( V \), defined as the discharge, \( Q \), divided by the cross-sectional area, \( A \), is used.

\[ V = \frac{Q}{A} \]  

(3.1)
Solution
The hinge provides force to support the weight and the hydrodynamic force of the system. This force can be computed from the momentum equation (Equation (3.7)).

\[ \sum F = \rho Q(\hat{V}_B - \hat{V}_A) \]

The hydrostatic forces are

\[ F_A = P_A A_A \hat{t} = \left(300,000 \times \frac{0.04}{4}\right) \frac{\pi (0.04)^2}{\pi} \hat{t} = 628.32 \hat{t} \]
\[ F_B = P_B A_B (-\cos 60 \hat{t} + \sin 60 \hat{j}) = 0 \]

and the velocities are

\[ \hat{V}_B = \frac{Q}{A_B} \left(\cos 60 \hat{t} - \sin 60 \hat{j}\right) = 31.83 \left(\frac{1}{2} \hat{t} - \frac{\sqrt{3}}{2} \hat{j}\right) \]
\[ \hat{V}_A = \frac{Q}{A_A} \hat{t} = 7.96 \hat{t} \]

so that

\[ 628.32 \hat{t} + F = (1000)(0.01)(15.92 \hat{t} - 27.57 \hat{j} - 7.96 \hat{i}) \]
\[ \hat{F} = -548.7 N \hat{i} - 275.7 N \hat{j} \]

PROBLEMS

3.3.1. A jet of water flowing freely in the atmosphere (x-direction) hits a curved vane and shoots straight up (z-direction). The velocity of the water jet is 10 ft/sec and has a 2-in. diameter. If the vane is assumed frictionless, determine the x and z forces exerted on the water jet by the vane.

3.3.2. At a firefighter’s convention, a certain competition pits two contestants in mock combat. Each is armed with a fire hose and a shield. The object is to push your opponent backward a certain distance with the spray. A choice of shields is offered. One shield is a flat garbage can lid; the other is a hemispherical lid that directs the water back toward your opponent. Which shield would you choose and why?

3.4 Energy Head in Pipe Flow

Water flow in pipes may contain energy in various forms. The major portion of the energy is contained in three basic forms:

1. kinetic energy,
2. potential energy,
3. pressure energy.

The three forms of energy may be demonstrated by using the general section of a pipe flow, as shown in Figure 3.3. The section of pipe flow approximately represents the concept of a stream tube that is a cylindrical passage with its surface everywhere parallel to the flow velocity; therefore, the flow cannot cross its surface.

Consider a control volume similar to that described in Figure 3.3. In time interval \( dt \), the water particles at section 1–1’ move to 1’–1” with the velocity \( V_1 \). In the same time interval, the particles at section 2–2’ move to 2’–2” with the velocity \( V_2 \). To satisfy the continuity condition,

\[ A_1 V_1 \, dt = A_2 V_2 \, dt \]

The work done by the pressure force acting on section 1–1’ in the time \( dt \) is the product of the total pressure force and the distance through which it acts, or

\[ P_1 A_1 \, ds_1 = P_1 A_1 V_1 \, dt \]

Similarly, the work done by the pressure force on section 2–2’ is

\[ -P_2 A_2 \, ds_2 = -P_2 A_2 V_2 \, dt \]

being negative because \( P_2 \) is in the opposite direction to the distance \( ds_2 \) traveled.

The work done by gravity on the entire mass of water in moving from 111’1” to 1’1”2’2” is the same as the work done if 111’1” were moved to the position 222’2” and the mass 1’1”22 was left undisturbed. The gravity force acting on the mass 111’1” is equal to the volume \( A_1 V_1 \, dt \) times the specific weight \( g = pg \). If \( h_1 \) and \( h_2 \) represent
By using Figure 3.8, Equations (1) and (2) are solved by iteration until both conditions are satisfied. The iteration procedure is demonstrated as follows:

Assume $f = 0.02$. From Equation (1) we get $V = 2.45$ m/sec; using this value in Equation (2) we get $N_p = 5.6 \times 10^6$. This number is taken to the Moody chart (Figure 3.8) to obtain the friction factor $f = 0.0122$, which is different from the assumed friction factor. For the second iteration, let $f = 0.0122$; we get $V = 3.14$ m/sec and $N_p = 7.2 \times 10^6$. The Moody chart gives $f = 0.0121$, which is considered close enough to the assumed value and will be used to calculate the flow velocity.

$$V = 3.15 \text{ m/sec}$$

This value and the cross-sectional area give the discharge.

$$Q = AV = \pi \left(\frac{3.14}{4}\right)(3.15) = 22.27 \text{ m}^3/\text{sec}$$

**Example 3.6**

Estimate the size of a uniform, horizontal welded-steel pipe installed to carry 14 ft$^3$/sec of water at 70°F (approximately 20°C). The allowable pressure loss is 17 ft/psi of pipe length.

**Solution**

The energy equation can be applied to two pipe sections 1 mi apart,

$$\frac{V_1^2}{2g} + P_1 + h_1 = \frac{V_2^2}{2g} + P_2 + h_2 + h_L$$

For a uniform, horizontal pipe with no localized head losses

$$V_1 = V_2$$

$$h_1 = h_3$$

$$h_L = h_f$$

and the energy equation reduces to

$$\frac{P_1 - P_2}{\gamma} = h_L = 17 \text{ ft}$$

From Equation 3.16

$$h_f = \frac{L}{2g} \frac{V_1^2}{2g} + \frac{L}{2g} \frac{Q^2}{2gD^4(4\pi)^2} + \frac{8\pi Q^2}{g^2h_f^3}$$

Therefore,

$$D^4 = \frac{8\pi Q^2}{g^2h_f^3} = 1530f$$

where $L = 5280$ ft, and $h_f = 17$ ft. At 20°C, $V = 1.08 \times 10^{-3}$ ft$^3$/sec. Assuming welded-steel roughness to be in the lower range of riveted steel, $e = 0.003$ ft, the diameter can then be found using the Moody chart (Figure 3.8) by means of an iteration procedure as follows.

Let $D = 2.5$ ft, then

$$V = \frac{Q}{A} = \frac{14 \text{ ft}^3/\text{sec}}{\pi (1.25 \text{ ft})^2} = 2.85 \text{ ft/sec}$$

and

$$N_p = \frac{VD}{V} = \frac{2.85(2.5)}{1.08 \times 10^{-3}} = 6.60 \times 10^6$$

$$e/D = 0.003 \text{ ft} = 0.00122$$

Entering these values into the Moody chart, we get $f = 0.021$. A better estimate of $D$ can be obtained by substituting the latter values into Equation (4), which gives

$$D = 0.021 \times 1530 \text{ ft} = 2.01 \text{ ft}$$

A new iteration provides $V = 4.46$ ft/sec $N_p = 8.3 \times 10^6$, $e/D = 0.0015$, $f = 0.022$, and $D = 2.0$ ft. More iterations will produce the same result.

**PROBLEMS**

3.5.1. 30-cm circular cast-iron pipeline, 2 km long, carries water at 10°C. What is the maximum discharge if a 4.6-m head loss is allowed?

3.5.2. A smooth concrete pipe (1.5-ft diameter) carries water from a reservoir to an industrial treatment plant 1 mi away and discharges it into a tank. The pipe begins 3 ft below the surface of the reservoir and runs downhill on a 1/100 slope. Determine the flow rate (in ft$^3$/sec) if the water temperature is 40°F (4°C) and the velocity at the outfall is negligible.

3.5.3. A horizontal, commercial steel (new) pipe, 1.5 m in diameter, carries 3.5 m$^3$/sec of water at 20°C. Calculate the pressure change in the pipe per kilometer length.

3.5.4. Two sections, $A$ and $B$, are 5 km apart along a 4-m riveted-steel pipe in its best condition. $A$ is 100 m higher than $B$. If the water temperature is 15°C and the pressure heads measured at $A$ and $B$ are 8.3 m and 16.7 m, respectively, what is the flow rate?

3.5.5. A 15-in. galvanized iron pipe is installed on a 1/50 slope (uphill) and carries water at 68°F (20°C). What is the pressure drop in the 65-ft-long pipe when the discharge is 18 cfs (ft$^3$/sec)?

3.5.6. The city water company wants to transport 1800 m$^3$/day of water per day to a plant some 8 km away. The water surface elevation at the reservoir is 5 m above the entrance of the pipe, and the pipe may be laid on a 1/500 slope. What is the minimum diameter of a concrete pipe that may be used if the water temperature varies between 5°C and 20°C? Assume pipe discharges into open air and the exit velocity is negligible.

3.5.7. Water flows through a hydraulically smooth pipe and the friction factor is 0.03. Determine the friction factor if the flow increases ten times.

3.5.8. Drawings to an old buried pipeline have been lost. At the entrance and exit of the pipe, two pressure gages measure a pressure drop of 37.6 ft. If the 6-in. galvanized iron pipe carries water at 68°F with a flow rate of 1.34 cfs (ft$^3$/sec), determine the length of the horizontal underground line.

3.5.9. Water flows through a 20-cm commercial steel pipe at 10°C. The flow rate is 80 l/sec and pressure is constant throughout the length of the pipe. Determine the slope on which it is laid.

3.5.10. Determine the flow rate of water at 10°C that will cause a pressure drop of 17,250 N/m$^2$ in 350 m of horizontal, cast-iron pipe, 60 cm in diameter.
3.6 Empirical Formulas for Friction Head Loss

Throughout the history of civilization, hydraulic engineers have designed and built systems to deliver water for people to use. In the earlier days, many of these designs were based on empirical formulas. Generally speaking, empirical design formulas were developed from the experiences of dealing with fluid flow under certain conditions in a specific range. Normally, they do not have a sound analytical basis. For this reason, the empirical formulas may not be dimensionally correct, and, if so, can only be applicable to the conditions and ranges specified.

One of the best examples is the Hazen-Williams formula, which was developed for water flow in larger pipes (D ≥ 5 cm, approximately 2 in.) within a moderate range of water velocity (V ≤ 3 m/sec approximately 10 ft/sec). This formula has been used extensively for the designing of water-supply systems in the United States. The Hazen-Williams formula, originally developed for the British measurement system, has been written in the form

\[ V = 1.318 C_{HW} R_h^{0.63} S^{0.54} \]  

(3.25)

where \( S \) is the slope of the energy gradient line ( EGL ), or head loss per unit length of the pipe, \( S = h/L, \) and \( R_h \) is the hydraulic radius, defined as the water cross-sectional area, \( A, \) divided by the wetted perimeter, \( P. \) For a circular pipe, \( A = \pi D^2/4 \) and \( P = \pi D, \) the hydraulic radius is

\[ R_h = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} \]  

(3.26)

The Hazen-Williams coefficient, \( C_{HW} \), is not a function of the flow conditions (i.e., Reynolds number). Its values range from 140 for very smooth, straight pipe down to 90 or 80 for old, unlined tuberculated pipe. Generally, the value of 100 is taken for average conditions. The values of \( C_{HW} \) for commonly used water-carrying conduits are listed in Table 3.2.

Note that the Hazen-Williams formula as shown in Equation (3.25) is applicable only for the British units in which the velocity is measured in feet per second and the hydraulic radius \( R_h \) is measured in feet. When used in S.I. units, the Hazen-Williams formula may be written in the following form:

\[ V = 0.85 C_{HW} R_h^{0.63} S^{0.54} \]  

(3.27)

where the velocity is measured in meters per second and the hydraulic radius, \( R_h \), is measured in meters.

The solution for the Hazen-Williams formula, Equation (3.27), may be obtained by direct computation or by the use of a solution chart (nomograph), as demonstrated in the following example.

(Note: Nomographs were very popular prior to the advent of the pocket calculator, which made exponential math calculations trivial. Nomographs are disappearing but are still found in some federal agency design manuals. The following example problems demonstrate their use. The actual nomographs have been relegated to Appendix B to denote their diminished status until they are phased out completely.)

<table>
<thead>
<tr>
<th>Pipe Materials</th>
<th>( C_{HW} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asbestos Cement</td>
<td>140</td>
</tr>
<tr>
<td>Brass</td>
<td>130-140</td>
</tr>
<tr>
<td>Brick sewer</td>
<td>100</td>
</tr>
<tr>
<td>Cast-iron</td>
<td></td>
</tr>
<tr>
<td>New, unlined</td>
<td>130</td>
</tr>
<tr>
<td>10 yr. old</td>
<td>107-113</td>
</tr>
<tr>
<td>20 yr. old</td>
<td>89-100</td>
</tr>
<tr>
<td>30 yr. old</td>
<td>75-90</td>
</tr>
<tr>
<td>40 yr. old</td>
<td>64-83</td>
</tr>
<tr>
<td>Concrete or concrete lined</td>
<td></td>
</tr>
<tr>
<td>Steel forms</td>
<td>140</td>
</tr>
<tr>
<td>Wooden forms</td>
<td>120</td>
</tr>
<tr>
<td>Centrifugetally spun</td>
<td>135</td>
</tr>
<tr>
<td>Copper</td>
<td>130-140</td>
</tr>
<tr>
<td>Galvanized iron</td>
<td>120</td>
</tr>
<tr>
<td>Glass</td>
<td>140</td>
</tr>
<tr>
<td>Lead</td>
<td>130-140</td>
</tr>
<tr>
<td>Plastic</td>
<td>140-150</td>
</tr>
<tr>
<td>Steel</td>
<td></td>
</tr>
<tr>
<td>Coal-tar enamel lined</td>
<td>145-150</td>
</tr>
<tr>
<td>New unlined</td>
<td>140-150</td>
</tr>
<tr>
<td>Riveted</td>
<td>110</td>
</tr>
<tr>
<td>Tin</td>
<td>130</td>
</tr>
<tr>
<td>Vitrified clay (good condition)</td>
<td>110-140</td>
</tr>
<tr>
<td>Wood stove (average condition)</td>
<td>120</td>
</tr>
</tbody>
</table>
Water Flow in Pipes  Chap. 3

Hydraulic Radius: \( R_h = A/P = \frac{A}{P} = \frac{0.00785}{0.514} = 0.025 \text{ m} \)

Energy Slope: \( S = h/L = \frac{24.6 \text{ m}}{200 \text{ m}} = 0.123 \)

Manning's Roughness Coefficient: \( n = 0.015 \)

Substituting the above quantities into the Manning equation, Equation (3.28), we have

\[
V = \frac{Q}{A} = \frac{1}{n} R_h^{1/2} S^{1/2} = \frac{1}{0.015} (0.00785)(0.025)^{1/2}(0.123)^{1/2} = 0.1517 \text{ m}^3/\text{sec}
\]

b. Solution Chart:

Applying the given conditions to the chart, Figure B2 (appendix), the following steps are suggested.

1. Locate the values of the energy slope, \( S = 0.123 \text{ m/m} \), and the Manning's roughness coefficient, \( n = 0.015 \), in the appropriate columns.
2. Connect the two points with a straight line and extend the line to the right to meet the "turning point" column.
3. Locate the pipe diameter, \( D = 0.1 \text{ m} \), on the middle column. Connect this point and the point on the "turning point" column with a straight line. Extrapolate the line to the left to obtain the discharge, \( Q = 0.015 \text{ m}^3/\text{sec} \); and extrapolate the line to the right to obtain the mean velocity, \( V = 2 \text{ m/sec} \).

PROBLEMS

3.6.1. A 6-km-long, new cast-iron pipeline carries 320 m/sec of water at 30°C. The pipe diameter is 30 cm. Compare the head loss calculated from (a) the Hazen-Williams formula, (b) the Manning formula, and (c) the Darcy-Weisbach formula.

3.6.2. A 2.5-ft riveted steel pipe carries water at 40°F from reservoir A to reservoir B. The length of the pipe is 2 m. The elevation difference between the two reservoirs is 335 ft. Calculate the discharge by (a) the Hazen-Williams formula, (b) the Manning formula, and (c) the Darcy-Weisbach formula.

3.6.3. Repeat the calculation in Problem 3.6.2 but use (a) the Manning formula and (b) the Hazen-Williams formula and compare the results with those obtained from the Darcy-Weisbach formula. Discuss the differences.

3.6.4. Two reservoirs 1200 m apart are connected by a 50-cm smooth concrete pipe. If the two reservoirs have an elevation difference of 5 m, determine the discharge in the pipe by (a) the Hazen-Williams formula, (b) the Manning formula, and (c) the Darcy-Weisbach formula. Compare solutions (a) and (b) with those found using the solution charts (Figures B1 and B2).

3.6.5. Research in the library and find, in addition to the three introduced in this section, six empirical formulas involving head loss in pipe lines. List the author(s) and limitations of each formula.

3.6.6. A buried concrete pipe (\( n = 0.012 \)) of unknown length needs to be replaced. The 2-ft conduit carries water between two reservoirs that differ in elevation by 15 ft. If the flow rate is 30 cfs (ft³/sec), how much new pipe is needed?

Sec. 3.7 Loss of Head Due to Contraction

3.7.1. Loss of head due to contraction between two reservoirs 25 km apart is 30 m.

(a) Compute the flow rate if a 30-cm commercial steel \( C_{me} = 0.49 \) pipeline connects the reservoirs. Compare your direct computation solution with that given by the solution chart.

(b) Compute the flow rate if a 20-cm commercial steel pipeline is used instead.

3.6.8. In Problem 3.6.7, use Manning's equation (commercial steel, \( n = 0.013 \), \( s = 0.045 \text{ m}^2/\text{sec} \)) to compute the flow rate. Compare the results from these two empirical formulas [Mannings, Equation (3.28), and Hazen-Williams, Equation (3.32)] to that of Darcy-Weisbach, Equation (3.16).

3.6.9. Based on Problem 3.6.2, determine the percent error in flow rate if the (a) estimate of \( C_{me} \) is off by 15%, and (b) the estimate of the \( n \) is off by 15%.

3.6.10. A concrete tunnel (\( n = 0.013 \)) with a semicircular cross section (radius = 1.0 ft) flows full with a discharge of 15 cfs (ft³/sec). What is the head loss in 1000 ft?

3.7 Loss of Head Due to Contraction

A sudden contraction in a pipe usually causes a marked drop in pressure in the pipe due to both the increase in velocity and the loss of energy to turbulence. The phenomenon of a sudden contraction is schematically represented in Figure 3.9.

The vertical distance measured between the energy gradient line (EGL) and the pipe centerline represents the total energy head at any particular location along the pipe. The vertical distance measured between the hydraulic gradient line (HGL) and the pipe centerline represents the pressure head, \( P_L \); and the distance between the EGL and HGL is the velocity head, \( V^2/2g \), at the location. After point B the HGL begins to drop as the stream picks up speed and a region of stagnant water appears at the corner of contraction C. Immediately downstream from the contraction the streamlines separate from the pipe wall and form a high-speed jet that reattaches to the wall at point E. The phenomenon that takes place between C and E is known as...
where \( v = 1.31 \times 10^{-6} \) at \( 10^\circ \text{C} \). For the 40-cm pipe, \( e/D = 0.00065 \), which yields \( f_1 = 0.0178 \). For the 20-cm pipe, \( e/D = 0.0013 \); so \( f_2 = 0.0205 \). Solving the above equation for \( V_2 \) provides the following:

\[
V_2^2 = \frac{1962}{11.36 + 156.25(0.0178) + 6000(0.0205)}
\]

\[
V_2 = 3.78 \text{ m/s}
\]

\[
V_1 = D/2(3.78) = 0.94 \text{ m/s}
\]

Hence,

\[
N_{Rk} = 3.05 \times 10^3(3.78) = 1.15 \times 10^4;
\]

\[
f_1 = 0.0183
\]

\[
N_{Rk} = 1.53 \times 10^3(3.78) = 5.78 \times 10^3;
\]

\[
f_2 = 0.021
\]

These \( f \) values do not agree with those stated previously, and a second trial must be made. For the second trial, we assume that \( K_e = 0.35, f_1 = 0.0183 \) and \( f_2 = 0.021 \). Repeating the above calculation, we get \( V_2 = 3.74 \), \( N_{Rk} = 1.14 \times 10^4 \), \( N_{Rk} = 5.72 \times 10^3 \). From Figure 3.8, we have,

\[
f_1 = 0.0183;
\]

\[
f_2 = 0.021.
\]

Therefore, the discharge is

\[
Q = A_2 V_2 = \frac{\pi}{4}(0.2)^2(3.74) = 0.117 \text{ m}^3/\text{sec}
\]

### PROBLEMS

3.10. A 75-m-long cast-iron pipe, 15 cm in diameter, takes water from a lake at 3 m below the lake surface. The pipe is laid on a 1/25 slope and has a 45° bend \( (R/D = 1.0) \) at midlength. What is the flow rate if the pipe discharges into a lake 5 m lower than the intake lake if the water temperature in the pipe is 20°C?

3.10.2. Determine the maximum discharge obtainable in a 3.5-ft steel penstock that brings water from a mountain reservoir to a hydroelectric powerplant. The entrance is 100 ft below the reservoir's water surface. The penstock is 1500 ft long, is laid on a 1/2 slope, and contains a globe valve.

3.10.3. A 34-m-high water tower supplies water to a residential area by means of a 20-cm-diameter, 800-m-long steel pipe. To preserve maximum pressure head at the delivery point, it is considered to replace 94% of the pipe length with a larger (30-cm-diameter) steel pipe connected by a 30° conusor. If the peak discharge demand is 0.10 m³/sec, find out how much pressure head it may gain at 20°C.

3.10.4. A cylindrical tank, 5 m in diameter is filled with water to a 3 m depth. A short horizontal pipe with a 20-cm diameter and a lift-type check valve is used to drain the tank from the bottom. How long does it take to drain 50% of the tank?

3.10.5. A pressure drop of 2.5 psi \((\text{lb/in})^2\) is measured across an abrupt expansion. Determine the flow rate if the pipe diameters are 8 in. and 16 in.

3.10.6. Water flows at a rate of 120 ft/sec in a pipe with a 30-cm diameter. Determine the contraction loss if the diameter is suddenly reduced to 15 cm. Compare with the head loss incurred when the 15-cm pipe suddenly expands to 30 cm.