Chapter 5: BJT AC Analysis

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BJT Transistor Modeling

- A model is an equivalent circuit that represents the AC characteristics of the transistor.

- A model uses circuit elements that approximate the behavior of the transistor.

- There are two models commonly used in small signal AC analysis of a transistor:
  - $r_e$ model
  - Hybrid equivalent model
BJT Transistor Modeling

Capacitors chosen with very small reactance at the frequency of application → replaced by low-resistance or short circuit.

Removal of the dc supply and insertion of the short-circuit equivalent for the capacitors.
BJT Transistor Modeling

Circuit redrawn for small-signal ac analysis
The $r_e$ Transistor Model

Common Emitter Configuration

![Transistor Circuit Diagram]

$Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$

$V_{be} = I_e r_e = (I_c + I_b)r_e = (\beta I_b + I_b)r_e = (\beta + 1)I_b r_e$

$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta + 1)I_b r_e}{I_b} = (\beta + 1)r_e \approx \beta r_e$
The $r_e$ Transistor Model  Common Emitter Configuration

\[ r_e = \frac{26 \text{ mV}}{I_E} \]
The output resistance $r$ is typically in the range of 40 kΩ to 50 kΩ.
Common-Base Configuration

\[ I_i = I_e \]

\[ I_o = -I_c \]

\[ V_i \]

\[ V_o \]

\[ I_c = \alpha I_e \]

\[ Z_i \]

\[ Z_o \]

\[ E \]

\[ C \]

\[ B \]
The output resistance $r_0$ is quite high. Typically extend into the megaohm range.

Common Base $r_e$ equivalent circuit
Common Emitter Fixed Bias Configuration

Common-emitter fixed-bias configuration.

Network after the removal of the effects of $V_{CC}$, $C_1$ and $C_2$. 
Substituting the $r_e$ model into the network.
Common Emitter Fixed Bias Configuration

Input impedance:
\[ Z_i = R_B \parallel \beta r_e \]
\[ Z_i \approx \beta r_e \quad \text{if} \quad R_E \geq 10\beta r_e \]

Output impedance:
\[ Z_o = R_C \parallel r_o \]
\[ Z_o \approx R_C \quad \text{if} \quad r_o \geq 10R_C \]

Voltage gain:
\[ V_o = -\beta I_b (R_C \parallel r_o) \]
\[ I_b = \frac{V_i}{\beta r_e} \]
\[ V_o = -\beta \left( \frac{V_i}{\beta r_e} \right) (R_C \parallel r_o) \]

\[ A_v = \frac{V_o}{V_i} = -\left( \frac{R_C \parallel r_o}{r_e} \right) \quad , \quad A_v = -\frac{R_C}{r_e} \quad \text{if} \quad r_o \geq 10R_C \]
Common Emitter Fixed Bias Configuration

\[ A_v = \frac{V_o}{V_i} = -\frac{(R_C||r_o)}{r_e} \]

Demonstrating the 180° phase shift between input and output waveforms.
Example 5.1

Determine $r_e$, $Z_i$ (with $r_o=\infty$), $Z_o$ (with $r_o=\infty$), $A_v$ (with $r_o=\infty$).
Repeat with $r_o=50$ kΩ.
Example 5.1 - Solution

\begin{center}
\begin{circuitikz}
\draw (0,0) to [v-] (2,0); \draw (2,0) to [2k\Omega] (4,0); \draw (4,0) to [10\mu F, inv] (2,2); \draw (2,2) to [T, \beta=100, \textcolor{red}{$I_o$}] (4,4); \draw (4,4) to [2k\Omega, \textcolor{red}{$V_o$}] (6,4); \draw (6,4) to [G] (8,4); \draw (8,4) to [10\mu F, \textcolor{red}{$Z_o$}] (6,2); \draw (6,2) to [G] (4,2); \draw (4,2) to [2k\Omega, \textcolor{red}{$V_i$}] (0,2); \draw (0,2) to [2k\Omega, \textcolor{red}{$Z_i$}] (0,0);
\end{circuitikz}
\end{center}
Common-Emitter Voltage-Divider Bias

The r_e model requires you to determine β, r_e, and r_o.
Common-Emitter Voltage-Divider Bias

Input impedance:

\[ R' = R_1 \parallel R_2 \]

\[ Z_i = R' \parallel \beta r_e \]

Output impedance:

\[ Z_o = R_C \parallel r_o \]

\[ Z_o \approx R_C \left| r_o \geq 10R_C \right. \]

Voltage gain:

\[ V_o = -\beta I_b \left( R_C \parallel r_o \right) \]

\[ I_b = \frac{V_i}{\beta r_e} \]

\[ V_o = -\beta \left( \frac{V_i}{\beta r_e} \right) \left( R_C \parallel r_o \right) \]

\[ A_v = \frac{V_o}{V_i} = -\frac{\left( R_C \parallel r_o \right)}{r_e} \]

\[ A_v = -\frac{R_C}{r_e} \left| r_o \geq 10R_C \right. \]
Example 5.2

Determine $r_e$, $Z_i$, $Z_o$ (with $r_o=\infty$), $A_v$ (with $r_o=\infty$). Repeat with $r_o=50 \, k\Omega$. 

![Circuit Diagram](diagram.png)
Example 5.2 - Solution

![Circuit Diagram]

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Common-Emitter Emitter-Bias Configuration

\[ I_i \rightarrow b \rightarrow c \rightarrow V_o \]

\[ \beta r_e \]

\[ \beta I_b \]

\[ V_i \rightarrow R_B \rightarrow b \rightarrow c \rightarrow V_o \]

\[ I_e = (\beta + 1)I_b \]

\[ Z_i \rightarrow R_B \rightarrow Z_b \]

\[ Z_o \rightarrow C_1 \rightarrow R_C \rightarrow V_o \]

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**Impedance Calculations**

**Input impedance:**

\[ V_i = I_b \beta r_e + I_e R_E \]

\[ V_i = I_b \beta r_e + (\beta + 1)I_b R_E \]

\[ Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta + 1)R_E \]

\[ Z_b \approx \beta r_e + \beta R_E = \beta \left( r_e + R_E \right) \]

\[ Z_b \approx \beta R_E \quad \text{for } R_E >> r_e \]

\[ Z_i = R_B \parallel Z_b \]

**Output impedance:**

\[ Z_o = R_C \]
Gain Calculations

Voltage gain:

\[ V_o = -I_o R_C = -\beta I_b R_C \]

\[ V_o = -\beta \left( \frac{V_i}{Z_b} \right) R_C \]

\[ A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b} \]

substituting \( Z_b \approx \beta (r_e + R_E) \)

\[ A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e + R_E} \]

and for the approximation \( Z_b \approx \beta R_E \)

\[ A_v = \frac{V_o}{V_i} \approx -\frac{R_C}{R_E} \]
Example 5.3  Without $C_E$ (unbypassed):

Determine $r_e$, $Z_i$, $Z_o$, $A_v$. Ignore $r_o$ for $r_o \geq 10(R_C+R_E)$.
Emitter-Follower Configuration

- This is also known as the common-collector configuration.
- The input is applied to the base and the output is taken from the emitter.
- There is no phase shift between input and output.
Impedance Calculations

Input impedance:

\[ Z_i = R_B \parallel Z_b \]
\[ Z_b = \beta r_e + (\beta + 1)R_E \]
\[ Z_b \approx \beta (r_e + R_E) \]
\[ Z_b \approx \beta R_E \quad \text{(for } R_E \gg r_e \text{)} \]
Impedance Calculations

Output impedance:

\[ I_b = \frac{V_i}{Z_b}, \quad I_e = (\beta + 1)I_b \]

\[ I_e = \frac{(\beta + 1)V_i}{\beta r_e + (\beta + 1)R_E} \]

\[ \sin c e \ (\beta + 1) \approx \beta \]

\[ I_e = \frac{V_i}{r_e + R_E} \]

To determine \( Z_o \), \( V_i \) is set to zero

\[ Z_o = R_E \parallel r_e, \quad Z_o \approx r_e \mid R_E \gg r_e \]
Gain Calculations

Voltage gain:

$$V_o = \frac{R_E}{R_E + r_e} V_i$$

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

$$A_v \approx 1 \quad R_E \gg r_e, \quad R_E + r_e \approx R_E$$
Example 5.7  
Determine $r_e$, $Z_i$, $Z_o$, $A_v$.
Example 5.7 - solution

\[ V_i \rightarrow I_i \]

\[ I_o \rightarrow Z_i \]

\[ \beta = 100, r_o = \infty \Omega \]

\[ 12 \text{ V} \]

\[ 220 \text{ k}\Omega \]

\[ 10 \mu\text{F} \]

\[ 10 \mu\text{F} \]

\[ 3.3 \text{ k}\Omega \]

\[ V_o \]