The Islamic University of Gaza
Faculty of Engineering
Civil Engineering Department
Water Resources Msc.

Groundwater Hydrology- ENGC 6301

lecture 2

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Semester: 1st/2010-2011
Aquifers and their Types

The amount of water yielded by a well, excavated through an aquifer,
- well diameter are inherent in the well itself.
- the permeability and the thickness of the aquifer are the most important.

Aquifers vary in depth, lateral extent, and thickness; but in general, all aquifers can classified into one of the two categories,

1. Unconfined or Non-artesian aquifers, and
2. Confined or Artesian aquifers
Unconfined Aquifers or Non-artesian Aquifers.

• The top most water bearing stratum, having no confined impermeable over burden (i.e. aquiclude) lying over it,

• The water level in wells of unconfined aquifer (gravity wells with a diameter of 2 to 5 m) will be equal to the level of the water table. Such wells are, therefore, also known as wells or
Confined Aquifers or Artesian Aquifers.

• Its upper and under surface by impervious rock formation *i.e.* aquicludes),

• A sufficient hydraulic head is created, it is 'called a *confined aquifer* or an *artesian aquifer*.

• A well excavated through such an aquifer, yields water that often flows out automatically, under the hydrostatic pressure, and

• May thus, even rise or gush out of surface for a reasonable height.

• The ground profile is high, the water may remain well below the ground level.
Confined Aquifers or Artesian Aquifers.

- A *flowing artesian well* has a pressure surface lies above the ground surface.
- A *non-flowing artesian well* has a pressure surface is below the ground surface, and will require a pump to bring the water to the surface, as
Perched Aquifers

*Perched aquifer is a special case which is sometimes found to occur within an unconfined aquifer*
Certain Other Important Terms

- **Specific capacity** of a well is the rate of flow from a well per unit of drawn down. It should be determined for the fall of the first meter, as it is not the same for all the drawdowns.

- **Coefficient of Storage (A)**

  is defined as the volume of water that an aquifer releases or stores per unit surface area of the aquifer per unit change in the component of head normal to that surface.
Certain Other Important Terms

- Coefficient of Storage (the change in its storage volume):

1. unconfined aquifer: this change can be determined by knowing the fall or the rise of the water table in a given time (t) and multiply it with the average specific capacity during this time,

2. confined aquifer: is equal to the volume of the water released from the aquifer of unit cross-sectional area and of the full height of the aquifer when the piezometric surface declines by unity.
Measurement of Yield of Underground Sources (Aquifers)

1. Estimation of the Yield by Estimation of the Velocity of Ground Water:

\[ Q = n \cdot v_a \cdot A \]

Where, \( A \) is the area of the aquifer opening into the well and \( v_a \) is actual flow velocity of groundwater.

- the velocity of the ground water flow (\( v_a \)) can be estimated by:
  - using Slichters or Hazen's empirical equations or
  - it can better be measured in the actual field by using chemical tracers, such as a dye; or by using electrical resistivity methods.
chemical tracers method

The time \((t)\) taken by a chemical tracer to travel a given known distance \((S)\) between two observation wells will directly indicate the ground flow velocity as

\[
v_a = \frac{S}{t}
\]

*This method is used to determine \(K\) such as*

\[
v = K \cdot I; \quad \text{or} \quad K = \frac{v}{I} = \frac{n \cdot v_a}{I} \quad \text{where} \quad I = \frac{H_L}{S}
\]

*Where \(H_L\) is the difference of water surface elevations of the two wells*
Pumping tests method

1. A well is, first of all, constructed through the aquifer, of which the yield is to be estimated.

2. Huge amount of water is drawn from the well, so as to cause heavy drawdown in its water level.

3. The rate of pumping is changed and so adjusted that the water level in the well becomes constant.

4. In this condition of equilibrium, the rate of pumping will be equal to the rate of yield,

5. and hence, the rate of pumping will directly give us the yield of the well, at a particular drawdown. -

Method will be detailed later on
Recuperating or Recovery test methods

1. water is first of all drained from the well at a fast rate, so as to cause sufficient drawdown.
2. The pumping is then stopped.
3. The water level in the well will start rising.
4. The rise is noted at regular intervals of time, till the initial level is reached.
5. Knowing the area of the well and the rise of the water level, the volume of the water yielded in that given time interval, can be worked out at different drawdowns.

These methods are also used to determine the characteristics of the aquifer.
- Dupuit, Thiem, and Theis developed the theoretical analysis of ground water flow, mainly used to compute $K$ and then $Q$

- We will first of all derive the equations of Thiem, and then switch to the equations of Dupuit,
Thiem's Equilibrium Formulas

1. Thiem's Formula for Unconfined Aquifer case

- Let a non-artesian well be driven, and water pumped heavily so as to cause sufficient draw-down.
- When the water level in the well decreases, the water level in the neighbourhood will also fall down; *(an inverted cone of depression)* all around the well,
- The base of this cone is a circle of radius \( R \), *(the circle of influence)*, and the inclined side is known as the *drawdown curve*. 

![Diagram showing an inverted cone of depression with a circle of influence and drawdown curve.]
Thiem's Equilibrium Formulas

1. Thiem's Formula for Unconfined Aquifer case

-two observation wells lying within the circle of influence of the main pumped well are to be driven.
-Let these wells be numbered as 1 and 2 (Fig. 16.9) and let them be at distances of $r_1$ and $r_2$ from the main well (centre to centre distance),
-Let $d$ be the depth of the well or the aquifer, below the static water table.
Thiem's Equilibrium Formulas

Let the main well be pumped at a sufficient rate, so as to cause heavy drawdown. Then, let the pumping be so adjusted that the equilibrium conditions are reached \((\text{the rate of pumping becomes equal to the rate of yield, and thus causing the water level to attain a constant value.})\) \textbf{equilibrium formula.}

Let \(S_1\) and \(S_2\) be the drawdowns in the two corresponding observation wells, at this equilibrium stage.
Thiem's Equilibrium Formulas

By Darcy's law

\[ Q = KIA \]

where \( I = \) Hydraulic gradient

Using cylindrical co-ordinates, we take \( r \) as the radius of any cylinder, and \( h \) as the height of the cone of depression at a distance \( r \) from the main well.

\[ I = \frac{dh}{dl} \approx \frac{dh}{dr} \]

the area of flow \( (A) \) is equal to \( 2\pi rh \)

\[ Q = KIA = K \cdot \frac{dh}{dr} \cdot 2\pi r \cdot h \]

\[ Q = 2\pi K h r \cdot \frac{dh}{dr} \]

\[ \frac{dr}{r} = \frac{2\pi K d h}{Q} \]

\[ \int_{r_1}^{r_2} \frac{dr}{r} = \int_{h_1}^{h_2} \frac{2\pi K}{Q} \cdot h dh \]

Actual velocity distribution

Assumed velocity distribution, i.e. uniform velocity or horizontal flow
Thiem's Equilibrium Formulas

\( K \) = Permeability of soil, which is assumed to be constant \textit{at all places and at all times} \textit{i.e.} (homogeneous soil), and in all directions, \textit{i.e.} (isotropic soil), by assuming the soil to be homogeneous and isotropic.

\[
\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi K}{Q} \int_{h_1}^{h_2} hdh
\]

\[
\log_e \frac{r_2}{r_1} = \frac{2\pi K}{Q} \left[ \frac{h_2^2}{2} - \frac{h_1^2}{2} \right]
\]

\[
\log_e \frac{r_2}{r_1} = \frac{\pi K}{Q} \left[ h_2^2 - h_1^2 \right]
\]

\[
Q \cdot \log_e \frac{r_2}{r_1} = \frac{K}{\pi \left( h_2^2 - h_1^2 \right)}
\]

\[
Q = \frac{\pi K \left( h_2^2 - h_1^2 \right)}{2.3 \log_{10} \frac{r_2}{r_1}}
\]

\[
K = \frac{Q \cdot \log_e \frac{r_2}{r_1}}{\pi \left( h_2^2 - h_1^2 \right)}
\]

\[
(h_2^2 - h_1^2) = (h_2 + h_1) \cdot (h_2 - h_1)
\]

\[
h_2 - h_1 = s_1 - s_2
\]
Thiem's Equilibrium Formulas

In the case of the amount of drawdown is small compared to the saturated thickness of the water bearing layer, we have:

\[ h_1 + h_2 \approx d + d = 2d \]

\[ (h_2^2 - h_1^2) \approx (s_1 - s_2) \times 2d = 2d (s_1 - s_2) \]

\[ Q \approx \frac{2\pi \cdot Kd (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}} \]

\[ Q \approx \frac{2\pi T (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}} \]

\( T \) is the transmissibility
Thiem's Equilibrium Formulas

The assumptions are as follows:

1. The aquifer is homogeneous, isotropic and of infinite and areal extent, so that its coefficient of transmissibility or permeability is constant everywhere.
2. The well has been sunk through the full depth of the aquifer and it receives water from the entire thickness of the aquifer.
3. Pumping has continued for a sufficient time at a uniform rate, so that the equilibrium stage or steady flow conditions have reached.
4. Flow lines are radial and horizontal, and flow is laminar.
5. The inclination of the water surface is small so that its tangent can be used in place of sine for the hydraulic gradient in Darcy's equation.
2. Thiem's Formula for Confined Aquifer case

-The formula used for the case of the unconfined aquifer has to be slightly modified in the case of an artesian aquifer,
-the flow is actually radial and horizontal.
-Rest of the assumptions remain the same and hold good in this case also

\[
Q = \frac{2\pi KH (h_2 - h_1)}{2.3 \log_{10} \frac{r_2}{r_1}}
\]

But \( h_2 - h_1 = s_1 - s_2 \)

Therefore, \( Q = \frac{2\pi KH (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}} \)

Also \( Q = \frac{2\pi T (s_1 - s_2)}{2.3 \log_{10} \frac{r_2}{r_1}} \)
Limitations of Thiem's Equilibrium Formulas

in actual practice,

-an aquifer is not fully homogeneous, or
-the well might have been dug half way through the aquifer, or
-permeability may not be uniform, or
-the ground water table may be inclined and thus, the base of the cone may not be a circle, or
-the equilibrium conditions might have not fully reached.

However, it is very difficult to assess the effects of these factors, and despite the various limiting assumptions,

Thiem's formula is widely used in ground water problems and many of its limitations are removed by appropriate adjustments.
Dupuit's Original Equilibrium Formulas

/no observation wells (as constructed in Thiem's formula) are constructed.

-The main well is pumped out so as to get sufficient drawdown,

-and then the rate of pumping is so adjusted as to establish equilibrium conditions (the water level in the well becomes constant).

-All the assumptions which have been made in the Theim's formulas hold good for the Dupuit's formulas also.
Dupuit's Original Equilibrium Formulas

- The only difference is that the integration which was done between the limits of \( r_1 \) and \( r_2 \) (radii of two observation wells) in Theim's formulas is changed,

- the integration is done between the limits \( r_w \) and \( R \), where

\[ r_w : \text{is the radius of the main pumped well and} \]
\[ R : \text{is the radius of influence.} \]

The radius of influence: is the distance from the centre of the pumped well to the point, where the drawdown is zero or is inappreciable.
Dupuit's Original Equilibrium Formulas

- Dupuit's Formula for Gravity Well or Unconfined Aquifer Case

\[ Q = K \cdot \frac{dh}{dr} \cdot 2\pi rh. \]

where \( K \) = Coefficient of permeability

\[ \frac{dr}{r} = \frac{2\pi K}{Q} \cdot h \cdot dh. \]

Integrating between the limits \( r_w \) and \( R \), we get

\[ \int_{r_w}^{R} \frac{dr}{r} = \frac{2\pi K}{Q} \int_{h_w}^{d} h \cdot dh. \]

\[ \log_e r \bigg|_{r_w}^{R} = \frac{2\pi K}{Q} \left[ \frac{h^2}{2} \right]_{h_w}^{d}. \]

\[ \log_e \frac{R}{r_w} = \frac{\pi K}{Q} \cdot [d^2 - h_w^2]. \]

\[ 2.3 \log_{10} \frac{R}{r_w} = \frac{\pi K}{Q} \cdot [d^2 - h_w^2]. \]
Dupuit's Original Equilibrium Formulas

- Dupuit's Formula for Gravity Well or Unconfined Aquifer Case

- \( R \) value is not easily assessable, various arbitrary values have been assigned to \( R \) by various investigators. Slitcher gives it as 150m and Tolman calls it as 300 m.

- when \( R = CQ \), where, \( C = \) is a constant and \( Q = \) is the discharge

\[
Q = \frac{\pi K \left( d^2 - h_w^2 \right)}{2.3 \log_{10} \left( \frac{CQ}{r_w} \right)}
\]

\( Q \) can be determined by trial and error method
Dupuit's Original Equilibrium Formulas

- Dupuit's Formula for Confined Aquifer Case

\[ Q = KIA \]
\[ Q = K \cdot \frac{dh}{dr} \cdot 2\pi \cdot rH \]
\[ \frac{dr}{r} = \frac{2\pi KH}{Q} \cdot dh \]

Integrating between \( r_w \) and \( R \), we get

\[ \int_{r_w}^{R} \frac{dr}{r} = \frac{2\pi K}{Q} \cdot H \int_{h_w}^{D} dh \]

or

\[ \log_e r \bigg|_{r=r_w}^{r=R} = \frac{2\pi K}{Q} \cdot H \bigg| h \bigg|_{h=h_w}^{h=D} \]

or

\[ 2.3 \log_{10} \frac{R}{r_w} = \frac{2\pi KH}{Q} \left[(D - h_w)\right] \]

\[ K = \frac{2.3 Q \log_{10} \frac{R}{r_w}}{2\pi H (D - h_w)} \]

\[ Q = \frac{2\pi KH (D - h_w)}{2.3 \log_{10} \frac{R}{r_w}} \]
Dupuit's Original Equilibrium Formulas

- Dupuit's Formula for Confined Aquifer Case

\[ Q = KIA \]

or

\[ Q = K \cdot \frac{dH}{dr} \cdot 2\pi \cdot rH \]

or

\[ \frac{dr}{r} = \frac{2\pi KH}{Q} \cdot dh \]

Integrating between \( r_w \) and \( R \), we get

\[ \int_{r_w}^{R} \frac{dr}{r} = \frac{2\pi K}{Q} \cdot H \int_{h_w}^{D} dh \]

or

\[ \log_e r \bigg|_{r=r_w}^{r=R} = \frac{2\pi K}{Q} \cdot H \left| h \right|_{h=h_w}^{h=D} \]

or

\[ 2.3 \log_{10} \frac{R}{r_w} = \frac{2\pi KH}{Q} \left[ (D - h_w) \right] \]
Dupuit's Original Equilibrium Formulas

- Dupuit's Formula for Confined Aquifer Case

\[
Q = KIA
\]

or

\[
Q = K \cdot \frac{dh}{dr} \frac{2\pi}{rH}
\]

or

\[
\frac{dr}{r} = \frac{2\pi KH}{Q} \cdot dh
\]

Integrating between \( r_w \) and \( R \), we get

\[
\int_{r_w}^{R} \frac{dr}{r} = 2\pi K \cdot \frac{D}{Q} \cdot H \int_{h_w}^{h} dh
\]

or

\[
\log_e r \bigg|_{r=r_w}^{r=R} = \frac{2\pi K}{Q} \cdot H \bigg|_{h=h_w}^{h=D}
\]

or

\[
2.3 \log_{10} \frac{R}{r_w} = \frac{2\pi KH}{Q} \left[ (D - h_w) \right]
\]

\[
K = \frac{2.3 Q \log_{10} \frac{R}{r_w}}{2\pi H (D - h_w)}
\]

\[
Q = \frac{2\pi KH (D - h_w)}{2.3 \log_{10} \frac{R}{r_w}}
\]