Problem 1:

a) Show that if $A = A_1 \cup A_2 \cup A_3 \ldots \cup A_m$ with $A_i \cap A_j = \emptyset$ for all $i \neq j$, then:

$$P[A \mid B] = P[A_1 \mid B] + P[A_2 \mid B] + P[A_3 \mid B] + \ldots + P[A_m \mid B]$$

[5 Marks]

b) A batch of 50 items contains 10 defective items. Suppose 10 items are selected randomly and tested. What is the probability that exactly 5 of the tested are defective?  

[6 Marks]

c) In an experiment A, B, C and D are events with probabilities 0.2, 0.35, 0.625 and 0.375, respectively. A and B are disjoint while C and D are independent. Find:

1- $P[A \cap B]$, $P[A \cup B]$, $P[A \cap B^c]$ and $P[A \cup B^c]$

2- Are A and B independent?

3- $P[C \cap D]$, $P[C \cap D^c]$ and $P[C^c \cap D^c]$

4- Are $C^c$ and $D^c$ independent?
Problem 2:

a) Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability $p$, independent of whether any other bit is received correctly. [15 Marks]

1- If the transmission continues until the receiving modem makes its first error, what is the PMF of $X$, the number of bits transmitted?

2- If the $p = 0.1$, what is the probability that $X = 10$? What is $P[X \geq 10]$?

3- If the modem transmits 100 bits, what is the PMF of $Y$, the number of errors?

4- If the probability of error is $p = 0.01$ and the modem transmits 100 bits, what is the probability of $Y = 2$ errors at the receivers? What is $P[Y \leq 2]$?

b) Suppose you know that the number of complaints coming into a phone centre averages 2.4 every ten minutes. Assume that the number of calls follows the Poisson distribution. [5 Marks]

1- What is the probability that there are three or fewer calls during the next 15 minutes?

2- What is the probability that there are exactly four calls during the next ten minutes?
Problem 3:
On the internet, data is transmitted in packets. In a simple model for the World Wide Web, the number of packets \( N \) needed to transmit a web page depends on whether the page has graphic images. If the page has Images (I), then \( N \) is uniformly distributed between 1 and 30 packets. If it has only text (T), then \( N \) is uniformly distributed 1 and 3 packets. Assuming a page has image with probability 0.4. Find:

1. \( P_{NI}(n) \)
2. \( P_{NT}(n) \)
3. \( P_N(n) \)
4. \( P_{N\leq7}(n) \)
5. \( E[N|N\leq7] \)
6. \( Var[N|N\leq7] \)
Problem 4:
The output voltage of a microphone (V) is a Gaussian (0, 5). V is the input of a limiting circuit with
cut-off value ±10V. A random variable, W is the output of the circuit:  [20 Marks]

\[
W = g(V) = \begin{cases} 
-10 & V < -10 \\
V & -10 \leq V \leq 10 \\
10 & V > 10 
\end{cases}
\]

1- Plot the input/output characteristic curve.
2- Find \( F_W(w) \).
3- Find \( P( W = w) \) at \( w = 10 \) and -10. [Use: \( \Phi(2) = 0.977 \)]
4- Find \( f_W(w) \)